Timely Persuasion

Deepal Basak, Zhen Zhou*

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Abstract

This paper proposes a simple dynamic information disclosure policy that eliminates panic. Panic occurs when some agents take an undesirable action (attack) because they fear that others will do the same, and thus, causing a regime change, even though it was not warranted. We consider a mass of privately informed agents who can attack a regime anytime within a time window. The attack is irreversible, the delayed attack is costly, and the delay cost is continuous. The policy we propose is called “disaster alert,” which at a given time publicly discloses whether the regime is going to change regardless of what the agents do. We show that a timely alert persuades the agents to wait for the alert and not attack if the alert is not triggered, regardless of their private information, and thus, eliminates panic. We apply this theory to demonstrate how forward-looking stress tests can help to stop bank runs.

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Introduction

In a game of strategic complementarity, often, an agent takes an undesirable action because he expects that others would do the same, even when it is not warranted. We call this panic. Imagine some investors who have made direct investments in an emerging market. Suppose an adverse shock hits the emerging economy. The investors have noisy information regarding the fundamental strength of the economy to face such a shock. If the fundamental is extremely poor, investors should exit the market. However, even if the fundamental is not extremely poor, because of strategic complementarity, investors could panic and start exiting the market if they think other investors will also exit. Can such panic be avoided? This paper proposes a simple dynamic information disclosure policy that eliminates panic.

The above example can be nicely captured through a canonical regime change game. A mass of agents decides whether to attack a regime or not. If the regime is strong enough to withstand the aggregate attack, the regime survives, otherwise the regime changes (or the status quo fails). There is a principal or an information designer who wants the regime to survive.

In practice, the attacks do not happen in one instance. For example, it takes time for investors to move the capital away from the emerging economy. They also may want to wait if they can get more information before finally deciding to exit. The existing literature usually assumes that agents move simultaneously. This is a reasonable assumption to capture the fact that agents do not know others’ actions while deciding their own. However, an information designer may explore the possibility of disclosing information about the history of play, so as to dissuade agents from attacking. To this end, we extend the canonical regime change game and allow the agents to attack within a time window. We assume that attacking is an irreversible action, and delayed attack is costly.

Going back to the capital outflow example, when an investor decides to exit a market, he may start talking to real estate agents, or firing employees. It is reasonable to assume that such exits are irreversible. The more investors start exiting, the less likely the regime will survive. However, since exiting takes time, the regime does not change right away. There is a time window $[0, T]$ such that the fate of the regime is decided at the end of the time window depending on the underlying strength of the regime and the aggregate attack. If an investor chooses to wait and exit later, he will lose the interest income from investing elsewhere. In this sense, a delayed attack is costly. Since the regime does not change in the
middle of the time window, it is reasonable to assume that the delay cost is continuous.\footnote{We will relax this assumption later when we consider financial markets. In a financial market, withdrawals could be immediate, and thus, the regime can change in the middle of the time window.}

The regulator can disclose information based on how many investors have started exiting. Note that exiting is an endogenous decision by the investors. Thus, unlike most of the existing literature on information design that focuses on disclosure about an exogenous state, we focus on disclosure about the endogenous history of attacks.

The agents are uncertain about the fundamental strength of the regime and get some noisy private signals about it. Unlike the global game literature, we do not impose independence on the noise. The noises may be independent or correlated. We allow for homogeneous or arbitrarily heterogeneous beliefs. Based on these signals, the agents form their beliefs about the fundamental and others’ signals. If an agent believes that the regime is not very likely to survive, he attacks. Many of the other agents may do otherwise, and the regime may survive. Thus, attacking right away could be a mistake ex-post. We assume that the information structure is such that the agents always believe that attacking could be such a mistake with positive probability, regardless of what others do. We refer to this assumption as \textit{Doubt}.\footnote{Note that an agent could also have doubts that waiting is a mistake. However, in this paper, we use the term doubt for this specific purpose only. This assumption is trivially true under standard global game information structure when the noise distribution has full support. If agents have heterogeneous beliefs, this assumption is not necessary. We will discuss this in Section 5.}

The principal commits to a dynamic information disclosure rule: At some time $t$, she will send a public message to the agents based on the exogenous fundamental and the endogenous history of the attack until time $t$. If the principal does not disclose any information, then the game is equivalent to a static regime change game. It is well known that under heterogenous beliefs, iterated elimination of never best responses leads to a unique cutoff strategy – the agents attack if and only if their private signals are below some threshold. This implies that if the fundamental does not warrant a run, but it is not strong enough, then many agents will attack, and the regime will not survive. Thus, panic can happen. On the other hand, if the principal adopts a full disclosure policy, then it is a complete information regime change game. Under complete information, if the fundamental is not so good that the regime will survive regardless of whether the agents attack, then there is always an equilibrium in which all the agents attack. We propose a simple partial disclosure policy called the “disaster alert.” A disaster alert at a pre-specified time $t$ is triggered if the regime is no longer strong enough to withstand any future attacks. Thus, a disaster alert sends an
early warning to the agents when it becomes evident that the regime cannot survive.

When the disaster alert is triggered, it is the dominant strategy to attack. On the other hand, if the disaster alert is not triggered, then the agents only learn that attacking is not the dominant strategy. However, if the other agents attack after seeing no alert, then the regime may fail. Thus, it is wise for an agent to attack if he anticipates that many of the other agents will attack even when the alert is not triggered. Interestingly though, if a sequentially rational agent has waited for the disaster alert, it must be that he will not attack when the alert is not triggered; otherwise, there is no positive option value of waiting to justify the cost of a delayed attack.

Thus, if the agents believe that others are sequentially rational, then once the principal discloses the disaster news, there is no strategic uncertainty left. Agents attack if and only if the alert is triggered. In other words, agents will follow the principal’s recommendation. However, since waiting is costly, an agent may not always wait for the disaster alert. In particular, an agent who receives a very low signal about the fundamental and believes that the disaster alert is very likely to be triggered may decide to attack based on his private signal rather than wait for the alert.

Since the agents who have waited for the alert will only attack if the alert is triggered, the regime survives for sure when the alert is not triggered. Hence, waiting for the alert and then following the recommendation avoids making a mistake of attacking a regime that will survive. In fact, under the doubt assumption, there is a positive probability that attacking right away is a mistake, regardless of what others do. This captures the benefit of waiting for the alert as compared to attacking immediately. However, waiting for the alert is costly. The principal can reduce this cost by setting the disaster alert at an earlier time. Since the cost of delay is continuous, a timely disaster alert policy guarantees that the expected benefit of waiting always outweighs the expected delay cost.

Thus, when the principal sets a timely disaster alert, the agents not only follow the principal’s recommendation after the alert but also always wait for the alert regardless of their signals. This implies that any regime that could have survived if no agent had attacked will indeed survive in the end. In other words, timely disaster alert eliminates panic.³

³The insight can be extended to the case where agents receive more information from outside sources over time. The extended policy in such a situation is to set the timely disaster alert right after the arrival of any new information, which could potentially induce a panic (See Section 3 for details).

In a coordination problem, the agents face uncertainty about the fundamentals as well as uncertainty about others’ strategies. However, if there is an early warning system in place
that sends a warning before a disaster happens, then the agents will wait for the warning, and all unnecessary panics can be avoided. This shows that panic is a fragile idea. If the agents are sequentially rational and they believe that others are sequentially rational,\(^4\) there is a simple way a principal can manipulate the agents and stop them from panicking. The principal does not need to know the private signals each agent receives. She uses a public disclosure policy, and she achieves the first best. More importantly, the policy does not violate the principal’s ex-post incentive compatibility. To see this, note that when the alert is triggered, the regime is doomed to fail regardless of what message the principal sends. This means the principal does not need an ex-ante commitment to implement such a policy.

After the financial crisis of 2008, bank supervisors start to adopt stress tests, partially as an information disclosure policy, in banking regulation.\(^5\) In practice, a stress test is designed to evaluate whether a bank or a financial institute is well equipped to deal with some stress scenarios that may arrive in the future. We apply our main result to design such forward-looking stress test that can eliminate bank run. To this end, in Section 4, we modify the benchmark setup and assume that the shock hits the bank or the financial institute at some time in a time window \([0, T]\) and not necessarily in the beginning. The agents do not know when precisely the shock will hit, but they have a common belief. In this setup, a timely disaster alert is a quick stress test that sends a warning to the investors if the bank’s balance sheet is so weak that regardless of what the investors do in the future, the bank cannot survive the shock when the shock arrives.

Note that unlike the slow-moving capital outflow, in a financial market, investors can withdraw immediately. Thus, in such a fast-moving market, a regime can change at any time in the middle of the time window. As soon as enough investor leaves, the regime may change, and thus, an investor may have to incur substantial cost if he waits. The effectiveness of such a stress test depends on when the shock hits. If the shock hits before the time of disclosure, it might be too late for investors to act on it. However, a timely stress test can limit this chance and ensure that this alert is very likely to be an early warning. Therefore, waiting for the alert is not likely to incur a significant loss, and thus, all agents would wait for the alert and follow the recommendation.

\(^4\)It is worth pointing out that we assume common belief in sequential rationality at the beginning of the game. Unlike the forward induction refinement, we do not impose any constraint on the beliefs conditional on unexpected histories. The agents are not “surprised” when they learn the disaster news.

\(^5\)“The plan aimed to impose transparency on opaque financial institutions and their opaque assets in order to reduce the uncertainty that was driving the panic.” – Timothy Geithner, Stress Test: Reflections on Financial Crises
The existing theory points out how a tough enough stress test can dissuade investors from running. In contrast, we focus on the timing aspect of stress tests. The disaster alert can be thought of as the weakest possible stress test in the sense that if the bank fails this test, the bank will inevitably fail regardless of what the investors do. We show that even the weakest stress test will dissuade the investors from running if it is conducted in a timely manner. In the process, unlike the existing results, this completely eliminates panic.

We assume that the agents are (sequentially) rational, and they believe others are (sequentially) rational. This is a much weaker assumption than the assumptions required for Nash equilibrium or any refinement of it. This assumption can be further weakened. We show that even if the agents believe that there is a small chance that others are not sequentially rational, and therefore, the regime may not survive even when the disaster alert is not triggered, the result is robust. In the absence of the doubt assumption, we need a stronger assumption – the common certainty of sequential rationality at the beginning of the game.

We consider a dynamic regime change game in which attacking is irreversible, and delayed attack is costly. These two features are essential. In many applications, the delay may not always be costly. For example, if investors earn flow returns, then by choosing the right time to exit, an investor may collect more flow returns by waiting. We construct such an example and show that a simple tax on the capital flight can make delayed exit costly, and thus, stops investors from panicking. One may expect that if the attack was reversible, then it will help the principal. However, when the attack is reversible, agents may decide to exit first and then come back if the alert is not triggered. This can cause a disaster alert to trigger in the first place.

**Related Literature**

This paper improves our understanding of how manipulating information can help in coordination. The most closely related papers are Goldstein and Huang (2016) and Inostroza and Pavan (2017). Similar to this paper, the papers mentioned above also consider a regime change game with privately informed agents. The authors consider an information designer who commits to an information disclosure rule. In a static regime change game, while Goldstein and Huang (2016) propose a simple stress test policy, Inostroza and Pavan (2017) design the optimal disclosure policy, which under some conditions can be a stress test. However, in general, the principal may benefit from discriminatory disclosure. In contrast, we consider a dynamic regime change game, in which the principal can disclose
information based on the past endogenous attacks.

Under a stress test, there can be multiple equilibria. But when the stress test is tough enough, Goldstein and Huang (2016) and Inostroza and Pavan (2017) argue that even in the worst equilibrium, i.e., the one in which agents attack most aggressively, they will not attack a regime that passes the stress test. Note that even if a run is not warranted, a bank may fail a stress test when the test is tough to pass. Thus, such a stress test does not eliminate panic. In contrast, since the disaster alert removes the strategic uncertainty after the alert, leveraging this, we show that a timely stress test eliminates panic.

Also, note that under a tough stress test, when the regime fails the test, the principal wants to lie to the agents. This violates the ex-post incentive compatibility. This means the principal needs ex-ante commitment power to implement a stress test policy. This is not the case with disaster alerts. If the regime is doomed to fail, the principal cannot do any better if she misreports.  

The crucial difference that drives this strong result is that the principal can disclose information about the past attacks. Since the agents endogenously choose the timing of their attack, the principal can exploit agents’ sequential rationality, and eliminate panic using a simple disclosure policy – a timely disaster alert. If the agents do not endogenously choose the timing of their attack, then the option value argument does not apply. Basak and Zhou (2019) consider this problem in which agents move sequentially in an exogenous order. The authors show that if the principal runs viability tests (weakest stress tests) sufficiently frequently, then the unique cutoff equilibrium involves no panic. Also, unlike this paper, the above result is limited to cutoff equilibrium, and it remains an open question whether this result can be extended to rationalizability.

This paper contributes to two strands of literature. First, the recent growing literature on dynamic information design. See Kamenica (2018) for a survey of the information design and Bayesian persuasion literature. Ely (2017) is the first paper to extend the static Bayesian persuasion problem of Kamenica and Gentzkow (2011) to a dynamic setting, but the author only considers history independent disclosure. Makris and Renou (2018) generalizes the correlated equilibrium of Bergemann and Morris (2016) to multistage game. While these papers assume that the players have noisy information about a payoff relevant state, Salcedo (2017) considers a complete information stage game but introduces

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6However, in practice, this may not always be the case. A principal may want to reduce attack even when she knows that a regime switch is inevitable. Also, the principal may have a time preference, e.g., she may want to defend the regime as long as possible. In these cases, implementing a disaster alert requires ex-ante commitment.
uncertainty regarding the game tree that governs the play. Doval and Ely (2019) consider a dynamic information design environment where the designer neither knows the private signals of agents, nor the game tree.

Finally, the paper contributes to the dynamic coordination game literature. To model panic, this paper borrows from the global game of regime change literature (See Morris and Shin (2003), Szkup (2017)). For a recent survey, see Angeletos and Lian (2017). Similar to Gale (1995), Dasgupta (2007) and Dasgupta, Steiner and Stewart (2012), we extend this canonical regime change game to allow for endogenous delay in attack.

The rest of the paper is organised as follows. Section 1 describes the model. Section 2 demonstrates that a timely disaster alert eliminates panic. Section 3 shows that the result can be extended to arrival of new information over time. Section 4 applies the theory to design forward looking stress test. Section 5 discusses which features on the model are essential and which features could be relaxed. Some of the proofs have been relegated to the appendix.

1 Model

**Players and Actions** The economy is populated by a principal, a continuum of agents, indexed by \( i \in [0, 1] \), and a regime. A shock hits the regime and it is commonly known. We normalize the time at which the shock hits as 0. Once the shock hits, the agents get a small time window \([0, T]\) to decide whether they want to attack the regime or not. Attacking can be taken as the action of exiting from a market, withdrawal of early investment, attacking a currency regime, making redemption from a mutual fund, etc.

Let us denote the action attack as 1 and not attack as 0. An agent \( i \) chooses \( a_i \in [0, T]^{\{0, 1\}} \) which describes whether he attacks or not at any time. Attacking is an irreversible action while not attacking is reversible. Hence, if agent \( i \) has already attacked by some \( t \), he has no more decision to make. However, if he has not attacked, then he has the option to attack at any time between \( t \) and \( T \), or not attack at all. If agent \( i \) decides to attack at time \( t_0 \in [0, T] \), then \( a_{it} = 0 \) for any \( t \in [0, t_0] \) and \( a_{it} = 1 \) for any \( t \in [t_0, T] \). For any agent \( i \) who decides to attack, i.e., \( a_{iT} = 1 \), let us denote the time of attack as \( t_i \equiv \min\{t \in [0, T] | a_{it} = 1\} \). The time of attack is defined to be \( t_i = \infty \) for an agent \( i \) who does not attack at all. Hence, \( a_i \) can simply be represented by the time of attack.
\( t_i \in [0, T] \cup \infty \). At any \( t \) within the time window, the mass of agents who already attack is

\[
N_t \equiv \int_{i \in [0,1]} 1\{i|t_i \leq t\} di.
\]

By definition, the mass of attacks \( N_t \in [0, 1] \) is (weakly) increasing in time \( t \).

**Fundamental States** The underlying state of the economy is captured by \( \theta \). We refer to it as the fundamental strength of the regime. It captures the preparedness of the regime to face the shock. At time 0, nature draws a state \( \theta \in \Theta \), where \( \Theta \) is a compact subset of \( \mathbb{R} \). It is common knowledge that \( \theta \) is drawn from some distribution \( \Pi \) with smooth density \( \pi \) strictly positive over \( \Theta \).

**Regime Outcome** Let \( r \in \{0, 1\} \) denotes the fate of the regime. We denote by \( r = 0 \) the event that the regime survives, and by \( r = 1 \) the complement event that the regime does not survive. The fate of the regime is decided at \( T \) depending on the fundamental state (\( \theta \)) and the aggregate attack until the end (\( N_T \)). The regime survives, i.e., \( r = 0 \), if, and only if \( R(\theta, N_T) \geq 0 \), where \( R(\cdot) \) is a continuous function that is increasing in \( \theta \) and decreasing in \( N_T \).

**Payoff** The agents are ex-ante identical and expected utility maximzers. If an agent does not attack \( (t_i = \infty) \), then he gets

\[
v(\theta, N_T) = \begin{cases} 
g(\theta, N_T) & \text{if } r = 0 \\
l(\theta, N_T) & \text{if } r = 1, \end{cases}
\]

and if he attacks at time \( t \), then he gets \( u(t) \). We normalize \( u(0) = 1 \). As is standard in the static regime change game,

\[
g(\theta, N_T) > 1 > l(\theta, N_T).
\]

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\(^7\)In Section 4, we relax this assumption when we consider a financial market. Since in a financial market, withdrawals can be instantaneous, the regime can change at any time \( t \) in the middle of the time window as soon as \( R(\theta, N_t) < 0 \).

\(^8\)See, for example, Inostroza and Pavan (2017).
This captures the fact that if the regime is going to survive \( r = 0 \), then not attacking is the desirable action, and if the regime is not going to survive \( r = 1 \), then attacking is the desirable action. We allow \( g(.) \) and \( l(.) \) to be non-monotonic in the arguments, but there exists \( g > 1 \) such that \( g(.) \geq g \) and there exists \( l < \bar{l} < 1 \) such that \( l \leq l(.) \leq \bar{l} \).

Different from the static payoff case, the agent has multiple opportunities to attack and delaying attack is costly, i.e., \( u(t) \) is decreasing in \( t \). Consider attack as exiting a market. Then, delaying exit means the investor is losing interest he could have earned by investing the money elsewhere. We assume that \( u(T) > \bar{l} \). This means that even at the last minute, if the agents learn that the regime will not survive \( (r = 1) \), attacking is the desirable action. We further assume that \( u(t) \) is Lipschitz continuous in \( t \).

**Dominance Region** There exists \( \bar{\theta}, \theta \in \Theta \) such that \( R(\theta, 0) = R(\bar{\theta}, 1) = 0 \). This means that when \( \theta \in \Theta^L = \Theta \cap (-\infty, \bar{\theta}) \), the regime cannot survive regardless of whatever strategy the agents take, and when \( \theta \in \Theta^U = \Theta \cap [\bar{\theta}, +\infty) \), the regime will always survive regardless of whatever strategy the agents take. We refer to \( \Theta^U \) (or \( \Theta^L \)) as the upper (or lower) dominance region where not attacking \( t_i = \infty \) (or attacking right away \( t_i = 0 \)) is the dominant strategy. We assume that \( \Theta^U \neq \emptyset \).

**Exogenous Information** In addition to the common prior \( \Pi \), each agent \( i \) receives a signal \( s_i \in \mathbb{R} \) about \( \theta \) before they decide when to attack (if at all). Given any underlying fundamental \( \theta \), the signal profile \( s(\theta) \in \mathbb{R}^{[0,1]} \) are drawn from a distribution \( F(s|\theta) \) with associated density \( f(s|\theta) \). Note that this allows for any arbitrarily correlated signals, ranging from independent private signals to public signals. We are interested in a short time window, in which the agents do not receive any more information about the fundamental or observe other agents’ actions.

For illustration, we sometime use the following information structure: common prior \( (\theta \sim U[\mathbb{R}]) \) and independent noise \( (s_i = \theta + \sigma \epsilon_i, \text{ where } \epsilon_i \sim F[-1/2, 1/2] \text{ and } \sigma > 0 \) scales the random noise \( \epsilon_i \)). We refer to this as the standard global game information structure.

**Principal** The principal’s payoff only depends on whether the regime survives or not. She gets 1 if the regime survives and 0 if it does not. The principal does not have access to the agents’ noisy private information.

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9It is easy to generalize to the case where the principal also wants to minimize aggregate attack conditional the regime’s survival.
Disclosure Policy For any \( \tau \in [0, T] \), the principal can disclose some information to the agents based on the exogenous fundamental \( \theta \) and the endogenous attack so far \( N_\tau \). We consider a continuous time model which means that an agent who has not attacked by time \( \tau \), gets the opportunity to attack again at \( (\tau + dt) \), where \( dt \to 0 \). At any time \( \tau \) we allow for a sequence of events to occur. Accordingly, we define \( \tau^- \) and \( \tau^+ \). First, at \( \tau^- \), an agent can attack, then at \( \tau^+ \), the principal can disclose information. There is no time discounting between \( \tau^- \) and \( \tau^+ \). Let \( S \) be a compact metric space defining the set of possible disclosures to the agents, and \( m_i(\tau, \theta, N_\tau) \in S \) be the message to agent \( i \). A general disclosure policy is \( \Gamma = (\pi, S) \) consists of the set of disclosed messages \( S \) and the disclosure rule \( \pi : [0, T] \times \Theta \times [0, 1] \to \Delta(S^{[0,1]}) \). The feature of endogenous move enables the principal to select the time of disclosure, and to make this information disclosure policy history dependent.

Robust Design We use rationalizability in extensive form game à la Pearce (1984) as our solution concept.\(^\text{10}\) Given a disclosure policy \( \Gamma \), let \( R(\Gamma) \) be the set of all possible rationalizable strategy profiles \( a \equiv (a_i(s_i)) \). Define

\[
\Theta^F(\Gamma) := \{ \theta \in \Theta | R(\theta, N_T(a)) < 0 \text{ for some } a \in R(\Gamma) \}.
\]

Thus, if \( \theta \notin \Theta^F(\Gamma) \), then the regime will survive regardless of whatever rationalizable strategies the agents play, and if \( \theta \in \Theta^F(\Gamma) \), then the regime may not survive. The principal’s objective is

\[
\min_{\Gamma} Pr(\theta \in \Theta^F(\Gamma)),
\]

where \( Pr(\theta \in \Theta^F(\Gamma)) = \int_{\Theta^F(\Gamma)} d\Pi \). That is, the principal anticipates, state by state, the “worst possible” outcome that is consistent with the agents playing some rationalizable strategy, and chooses the policy \( \Gamma \) to minimize the ex-ante chance that the regime may not survive.

Note that when \( \theta \in \Theta^L \), the regime fails irrespective of the size of the attack. Hence, any disclosure policy \( \Gamma \) cannot endure such a regime, i.e., \( \Theta^L \subseteq \Theta^F(\Gamma) \). A regime could also fail even when it is not warranted \( (\theta \notin \Theta^L) \) because the agents attack thinking that others will attack. We refer to this as panic-based attacks. Let us define \( \Theta^P(\Gamma) := \Theta^F(\Gamma) \setminus \Theta^L \) for any policy \( \Gamma \) as the set of fundamental in which the regime can fail because of panic-

\(^{\text{10}}\) We only assume that the agents commonly believe in sequential rationality in the beginning. We do not impose any restriction on beliefs conditional on unexpected histories.
based attacks. If $\Theta^P(\Gamma) = \emptyset$, then we say that the policy $\Gamma$ eliminates panic.

2 Main Result

We restrict our attention to a simple information disclosure policy. We show that, under some reasonable assumption on agent’s exogenous information structure, this simple disclosure policy eliminates panic. In other words, this policy induces the agents to perfectly coordinate their actions and never attack a regime when it is not warranted ($\theta \notin \Theta^L$).

Disaster Alert

We refer to the following disclosure policy as disaster alert. The principal only discloses information once at some time $\tau \in [0, T]$. The public signal $d^\tau$ is generated based on the underlying fundamental $\theta$ and the history of attacks $N_\tau$ as follows

$$d^\tau(\theta, N_\tau) = \begin{cases} 
1 & \text{if } R(\theta, N_\tau) < 0 \\
0 & \text{otherwise}.
\end{cases}$$

We denote this binary public disclosure policy as $\Gamma^\tau$. Upon receiving the signal $d^\tau = 1$, agents understand the regime cannot survive in the end, i.e., $R(\theta, N_T) < 0$ (Since $N_t$ is weakly increasing in $t$). Hence, for agents who have not attacked, it is the dominant strategy to attack at time $\tau + dt$. In this sense $d^\tau = 1$ acts as an alert for disaster. On the other hand, if the alert is not triggered, or $d^\tau = 0$, agents understand that $R(\theta, N_\tau) \geq 0$, and thus, the regime will survive if no agent attacks the regime after time $\tau$. In the spirit of Bayesian persuasion, this can be thought of as the principal sending a recommendation at time $\tau$ to the agents to “attack” when the disaster alert is triggered, and “not attack” otherwise.

Option Value of Waiting

Under the policy $\Gamma^\tau$, agents will only have one chance to get new information at $\tau^+$. Hence, attacking at any time $t_i \in (0, \tau^-)$ is dominated by attacking at time $t_i = 0$ since delayed attack is costly. Similarly, after receiving the new information $d^\tau$, attacking at anytime $t_i \in (\tau + dt, T]$ is dominated by attacking immediately after the disclosure at $\tau + dt$. 

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Lemma 1 (Option Value) Under the disclosure policy $\Gamma^\tau$, for any noisy signal $s_i$, the only rationalizable strategies are

$A$: attack at time 0, i.e., $a_{it}(s_i) = 1$ for all $t \in [0, T]$, and

$W$: wait until time $\tau$ for the disaster alert, i.e., $a_{it}(s_i) = 0$ for all $t \in [0, \tau + dt)$, and then follow the principal’s recommendation $a_{it}(s_i, d\tau) = d\tau$ for $t \in [\tau + dt, T]$.

Proof. First, it is not rational for agents to attack at any time other than 0 and $\tau + dt$. Secondly, when $d\tau = 1$, the dominant action is attack. Hence, the only possible strategies are: $A$, $W$ and attacking at time $\tau + dt$ independent of $d\tau$. Let us call this third strategy $W'$. Note that the strategy of $W'$ generates a payoff of $u(\tau + dt)$, which is strictly less than $u(0)$. Thus, it is strictly dominated by $A$. ■

For any agent who decides to wait for the disclosure (instead of attacking immediately), the information that will be disclosed at time $\tau$ must be valuable to him. This means that he will never take the same action (attack) regardless of the news disclosed in future. Otherwise, there is no option value associated with the information arriving in the future and hence he will not wait. Chamley and Gale (1994) and Gul and Lundholm (1995) made a similar argument in the context of social learning in which an agent can learn from others’ actions, but such actions do not affect his payoff. The intuition is simple - consider two agents deciding whether to attack at time 1 or time 2. If an agent waits to see whether the other agent attacks or not, it must be that he will take different actions conditional on whether the other agent attacks at time 1 or not. Otherwise, there is no positive option value of waiting.

In this paper, the principal controls the information flow after time 0. If the disaster alert is triggered ($d\tau = 1$), then attacking is the dominant strategy for an agent. Therefore, for positive option value of waiting, it must be that the agent will not attack when the alert is not triggered.

This implies that when the disaster alert is not triggered ($d\tau = 0$), the regime will survive in the end. This is because there is no further attack after time $\tau$ when $d\tau = 0$, i.e., $N_T = N_\tau$. For that reason, no alert ($R(\theta, N_\tau) \geq 0$) implies the survival of the regime, i.e., $R(\theta, N_T) \geq 0$. Consequently, under the policy $\Gamma^\tau$, the agents who decide to wait for the information disclosure, perfectly coordinate their actions. This completely removes the strategic uncertainty after time $\tau$ since the agents understand perfectly what other agents would do after getting the new information.
This is in sharp contrast with the static regime change game, in which the agents move simultaneously. In a static regime change game, the strategic uncertainty cannot be removed by publicly disclosing that attacking is not the dominant strategy. To see this, consider a disaster alert before the agents make any decision. This alert is triggered if $\theta < \theta$. If this alert is not triggered, then the agents know that $\theta \geq \theta$, or the regime will survive if no agent attacks. When this is publicly known, one possible equilibrium outcome is that no agent attacks and the regime survives. However, this is not the unique rationalizable strategy. If an agent receives a low signal and believes others will attack, then he will attack as well. In fact, Angeletos, Hellwig and Pavan (2007) show that there are many other possible equilibria in which the regime could fail because of panic-based attacks. However, under endogenous timing, if an agent with low signal decides to attack even after the disaster alert is not triggered ($d^r = 0$), then he is better off not waiting for the alert at all. So, it follows from Pearce (1984)'s extensive form rationalizability that either an agent attacks right away, or waits for the alert, and if he waits for the alert, then he follows the principal’s recommendation afterwards. However, it is possible that the agents do not wait for the alert and attack right away, and by doing so, they trigger the alarm.

**Reasonable Doubt**

The following assumption restricts the information generating process $F$. It says that regardless of whatever noisy signal an agent receives, he always assigns some positive chance that $\theta \in \Theta^U$. In other words, he always has some doubt that attacking is a mistake regardless of what other agents do.

**Assumption 1** *(Doubt)* There exists $\varepsilon > 0$ such that, any agent $i$ with noisy signal $s_i$ believes that

$$
P(\theta \in \Theta^U | s_i) = \frac{\int_{\theta \in \Theta^U} f_i(s_i|\theta)\pi(\theta)d\theta}{\int_{\theta \in \Theta} f_i(s_i|\theta)\pi(\theta)d\theta} > \varepsilon,
$$

where $f_i(s_i|\theta) = \text{marg}_{s_{-i}} f(s|\theta)$.

In particular, if $f_i$ has full support and it is bounded away from 0, then the above assumption holds true. $^{11}$

$^{11}$A sufficient condition that validates this doubt assumption is that $f_i(s_i|\theta) > \epsilon \pi(\theta^U)$ for all $i \in [0, 1]$, $s_i \in \mathbb{S}$ and $\theta \in \Theta^U$.  

13
**Timely Disaster Alert**

**Lemma 2 (Timely Alert)** There exists $\hat{\tau} > 0$, such that under the disclosure policy $\Gamma^\tau$, where $\tau < \hat{\tau}$, for any signal structure satisfying Assumption 1, the only rationalizable strategy for an agent with signal $s_i$ is $W$, i.e., wait for the disaster alert and then follow the principal’s recommendation.

**Proof.** Consider an agent who has decided to wait for the disclosure (plays $W$). If $R(\theta, N_r) < 0$, the alert triggers ($d^\tau = 1$). Then, he attacks at time $\tau + dt$ and gets $u(\tau + dt)$. Otherwise, $R(\theta, N_r) \geq 0$ and the alert does not trigger ($d^\tau = 0$). We know from Lemma 1 that the agent will not attack. Thus, the expected payoff from playing $W$ is

$$\mathbb{P}(d^\tau = 1|s_i)u(\tau + dt) + \mathbb{P}(d^\tau = 0|s_i)E(v(\theta, N_T)|s_i, d^\tau = 0).$$

If $d^\tau = 0$, it follows from Lemma 1 that no agent who has waited will attack, i.e., $N_T = N_r$. This implies that when the alert is not triggered ($d^\tau = 0$), $R(\theta, N_T = N_r) \geq 0$, i.e., the regime survives ($r = 0$). Recall that if $r = 0$, then by not attacking the agent gets $g(\theta, N_T = N_r)$. Finally, since $dt \to 0$, the expected payoff from playing $W$ simplifies to

$$\mathbb{P}(d^\tau = 1|s_i)u(\tau) + \mathbb{P}(d^\tau = 0|s_i)E(g(\theta, N_T = N_r)|s_i).$$

While, the expected payoff from attacking immediately ($A$) is $u(0) = 1$. Hence, the expected payoff difference from strategy $W$ as compared to $A$ is

$$D(\Gamma^\tau, s_i) = \mathbb{P}(d^\tau = 1|s_i)(u(\tau) - u(0)) + \mathbb{P}(d^\tau = 0|s_i)(E(g(\theta, N_r)|s_i) - 1). \quad (1)$$

Since $u(t)$ is Lipschitz continuous, $u(0) - u(\tau) \leq K\tau$ for some positive finite $K$. It follows from Assumption 1 that $\mathbb{P}(d^\tau = 0|s_i) > \epsilon$. Also, recall that $g(\theta, N) \geq g > 1$. Therefore,

$$D(\Gamma^\tau, s_i) > -(1 - \epsilon)K\tau + \epsilon(g - 1).$$

Define

$$\hat{\tau} := \frac{\epsilon}{1 - \epsilon} \left(\frac{g - 1}{K}\right).$$

If $\tau < \hat{\tau}$, then $D(\Gamma^\tau, s_i) > 0$ for any $s_i$. This implies an agent prefer to wait for the alert and then follow the principal’s recommendation ($W$) rather than attacking right away ($A$), regardless of his private signal. ■
The tradeoff agents face when choosing between attacking immediately ($\mathcal{A}$) and the strategy of wait and see ($\mathcal{W}$) is as follows. A cost $(u(0) - u(\tau))$ (lost interest income) is associated with a delayed attack when the alert ($d^\tau = 1$) is triggered. While, taking the strategy of wait and see can prevent agents from making a mistake by moving early, i.e., attacking a regime that survives. The benefit from this more informed choice is at least $(g - 1)$. Assumption 1 guarantees that regardless of the signal $s_i$, an agent assigns positive probability that the alert will not be triggered regardless of what other agents do, i.e., attacking is definitely a mistake. Because of that, the benefit from waiting is strictly positive regardless of what other agents would do. Lemma 2 shows that, if the disaster alert can be set in a timely manner, the cost of delay will be limited (strictly lower than the expected benefit), which makes the strategy of wait and see ($\mathcal{W}$) a strict dominant strategy.

We use the solution concept of extensive form rationalizability. However, the argument only use the fact that an agent is sequentially rational and he believes that others are sequentially rational. It does not require higher order beliefs such as an agent believes that others believe that he is sequentially rational and so on. As long as an agent is sequentially rational, the option value argument holds (Lemma 1), i.e., an agent will take either $\mathcal{A}$ or $\mathcal{W}$. If he believes that others are sequentially rational, he understand that the regime will not fail if the disaster alert is not triggered.\[12\] For a timely disaster alert, under the assumption 1, the expected payoff from $\mathcal{W}$ is strictly higher than $\mathcal{A}$, regardless of whether others play $\mathcal{W}$ or $\mathcal{A}$. Thus, if an agent believes that others are rational, he will take the strategy $\mathcal{W}$ rather than $\mathcal{A}$.

**No Panic**

**Theorem 1 (No Panic)** Under the disclosure policy $\Gamma^\tau$ with $\tau < \hat{\tau}$, if the information structure satisfies Assumption 1, then there is no panic, i.e.,

$$\Theta^ F(\Gamma^\tau) = \emptyset.$$

**Proof.** The only rationalizable strategy is $\mathcal{W}$ when $\tau < \hat{\tau}$ (Lemma 2). Thus, $N^\tau = 0$. For any regime with $\theta \not\in \Theta^ L$, $R(\theta, N^\tau) \geq 0$. Hence, no alert will be triggered and no further attack happens, or $N^ T = N^\tau = 0$ (Lemma 1). Therefore, any regime with $\theta \not\in \Theta^ L$ survives (since $R(\theta, N^ T) \geq 0$). Hence, $\Theta^ F(\Gamma^\tau) = \Theta^ L$ and $\Theta^ P(\Gamma^\tau) = \emptyset$. \[12\]

\[12\] If he does not believe that all the other agents are sequentially rational, then this argument does not hold. We will discuss this issue in Section 5.
Theorem 1 follows immediately from Lemma 1 and Lemma 2. Under a timely disaster alert, all agents, regardless of their noisy signal, would wait for the disclosure and follow the recommendation of the principal afterwards. Since all the agents are waiting, any regime that can survive without attack ($\theta \notin \Theta^L$) will not trigger the alert. Since the alert is not triggered, agents will follow the principal’s recommendation and not attack. Thus, any regime that can survive without any attack, will survive for sure. In other words, timely disaster alert eliminates panic. Note that this simple public disclosure policy makes the principal’s favorite outcome the unique rationalizable outcome.

Under strategic complementarity, “runs” are common. Runs could be based on fundamental ($\theta \in \Theta^L$), but it could also be because of panic ($\theta \in \Theta^P$). The principal cannot save a regime when run is based on fundamental, but the above theorem shows that panics can be eliminated. More importantly, it can be eliminated using a very “simple” disclosure policy. Recall that the principal does not have access to the agents’ private signals. The disaster alert policy does not require disclosure conditional on such private signals. Moreover, the policy is a binary and public disclosure – whether the alert is triggered or not, and it does not send private messages to the agents. Finally, the alert is triggered only when $R(\theta, N_r) < 0$, i.e., the regime cannot survive regardless of what the agents do. This means the principal cannot get a higher payoff by misreporting. Thus, the principal’s ex-post incentive compatibility holds.

3 New Information and Repeated Disaster Alert

So far, we focus on a short time window $[0, T]$ in which the agents could react to a bad news. In contrast to the simultaneous move game, we show that the principal can exploit this endogenous timing and stop agents from panicking. The insight has nothing to do with the length of the time window. But if the time window is not small, we may expect that new information may arrive over time. For example, if the time window is a month, the agents may receive weekly updates regarding the fundamental. In this case, the agents cannot be certain that even if the timely disaster alert is not triggered, agents will not panic later and attack when they receive new information. This could make the disaster alerts ineffective. Here, we consider the optimal disclosure policy under exogenous arrival of new information.

We assume that information does not arrive very frequently. In fact, there are only finitely many signals that can arrive in the time interval $[0, T]$. Suppose that (some) agents
have multiple chances of getting outside information at $t_m (m = 0, 1, \ldots, M)$, where $t_0 = 0$. This gives the principal the scope to set timely disaster alert after some agents receive news at time $t_m$ and before they receive news again at $t_{m+1}$. The doubt assumption is generalized as follows.

$$\exists \varepsilon > 0 : P(\theta \in \Theta | s^0_i, s^1_i, s^2_i, \ldots, s^M_i) > \varepsilon \text{ for any } s^0_i, s^1_i, s^2_i, \ldots, s^M_i.$$  

With new arrival of exogenous information, a natural extension of the one-shot disclosure policy is to set the disaster alert right after any new information arrives. To reduce burden of notation, we use the same notation $\Gamma^\tau$ to capture this modified policy in which there are $(M + 1)$ disaster alerts at time $(t_m + \tau)^\tau$ for $m = 0, 1, \ldots, M$.

**Theorem 2** When new information arrives over time, under the extended disclosure policy $\Gamma^\tau$ with $\tau < \hat{\tau}$, there is no panic.

To understand the argument consider the simple case – $M = 1$. Suppose agent $i$ receives a noisy private signal $s^0_i$ at time $0^-$ (as before) and $s^1_i$ at time $t_1^-$ for some $t_1 \in (0, T)$ about the fundamental $\theta$. The agents can act based on the private signal they receive as early as in the same period, i.e., at time 0 and $t_1$. We maintain the same assumptions regarding the fundamental state and the noisy information.

The basic argument is an extension of our main result. First of all, if the first alert has been triggered at time $\tau$, i.e., $d^\tau = 1$, then all agents have attacked. Thus, there is no need to think about any decision making after the new information arrives. Let us assume otherwise, and start our analysis from time $t_1$ onwards. Consider any agent who has not attacked before time $t_1$ and receives the new information. Based on the same argument as in Lemma 1, for any possible signal $s^0_i$ and $s^1_i$, agent $i$ either attacks at time $t_1$, or waits for the second alert and then attacks if and only if $d^{t_1+\tau} = 1$.

Under the extended doubt assumption, the agents cannot be sure that the regime will fail if all others attack after getting more exogenous information. Therefore, we can apply the same argument as in Lemma 2 to show that, with a timely disaster alert ($\tau < \hat{\tau}$), any agent who does not attack (after seeing no alert being triggered at time $\tau$, or $d^\tau = 0$), will wait for the next disclosure ($d^{t_1+\tau}$) and only attack when the second alert is triggered ($d^{t_1+\tau} = 1$).

Hence, if the first alert is not triggered ($d^\tau = 0$), no one who has waited will act on the new information, but would wait for the second alert and follow the recommendation. In
this case, \( d^{1+\tau} = d^\tau = 0 \). Otherwise, if the first alert is triggered, all agents who have not attacked would attack immediately and thus the second alert becomes irrelevant. Hence, given the timely disaster alert in the future can always prevent agents from acting on any new information, the problem faced by agents is essentially the same as the one in the benchmark setup. Thus, the extended timely disaster alert policy eliminates panic.

4 Forward-looking stress test

The U.S. government responded to the aftermath of the financial panic during the great recession in 2008 with various measures such as liquidity injection and debt guarantees. Stress tests which involve information production and disclosure emerged as a potent tool to quell panic during times of economic uncertainty. One of the most important functions of the stress tests is to provide credible information about how banks are likely to perform under severely distressed macroeconomic conditions. In practice the regulators collect data from the banks and based on their models investigate how the banks’ balance sheet perform under different kinds of stress scenarios which may arise in the future.

In practice, there is evidence that the U.S. stress tests produced credible information about the financial institutions and helped restore confidence in the banking system.\(^{13}\) The contemporaneous stress tests in Europe were not as successful. While the tests in 2009, 2010 did not have noticeable impact, what truly stands out is the failure of Dexia, a bank that had to be bailed out three months after it had passed the 2011 stress test (See Acharya, Engle and Pierret (2014)). Anderson (2016) argues that the reason behind this contrasting experiences is that while the stress test attempt in the U.S. was ‘timely’, it was perhaps, ‘too little, too late’ in the Europe.\(^{14}\)

We look into the forward-looking feature of stress tests and investigate not only how much to disclose, but also the timing of such disclosure.\(^{15}\) As in our benchmark setup,

\(^{13}\) Peristiani, Morgan and Savino (2010) document evidence to show that stress tests helped quell the financial panic by producing vital information about banks. Bernanke (2013) admits “Supervisors’ public disclosure of the stress tests results helped restore confidence in the banking system...” Gorton (2015) states that the tests results were viewed as credible and the stress tests are widely viewed as a success.

\(^{14}\)In the US, the plan for stress testing was announced on Feb 10, 2009. The white paper describing the procedures employed in SCAP was released on Apr 24, 2009 and the results of SCAP were disclosed on May 7, 2009. On the other hand, although signs of instability in the financial system became apparent around the same time as the US, the first European stress tests were conducted in October 2009.

\(^{15}\)In practice, a stress test policy is also supplemented by other measures of banking regulation which will not be a part of our model. For examples of such measures, see Faria-e Castro, Martinez and Philippon (2016) for fiscal capacity; see Shapiro and Skeie (2015) for the cost of injecting capital; see Orlov, Zryumov and
we assume that nature chooses the bank’s preparedness $\theta$ at time 0, and the agents receive some noisy information about $\theta$ at time 0. However, we introduce a new uncertainty regarding when the shock hits the bank or the financial institute. Moreover, we modify the setup to capture the fast-moving feature of the financial market. Formally, we make two modifications to our benchmark setup.

1. The shock hits the bank or the financial institute stochastically following a distribution $G(t)$, with atomless density, $G(0) = 0$, and $G(T) = 1$. Before the shock hits, the fundamental is $\theta_t = \theta_0 > \bar{\theta}$. After the shock hits $\theta_t = \theta$.

2. The regime can switch at any time $t \in [0, T]$ as soon as $R(\theta, N_t) < 0$. If the regime has changed by time $t$, we say $r(t) = 1$, otherwise $r(t) = 0$.

Recall that in our the benchmark setup the shock hits in the beginning. In contrast, we are now assuming that the shock will hit at some time in the time window $[0, T]$. Until the shock hits, $R(\theta_t, N) \geq 0$ for any $N$, i.e., withdrawal does not cause a regime change or a bank failure. However, if the bank is not well prepared (low $\theta$), and agents start attacking (high $N_t$), then the bank can fail when the shock arrives. The second modification is to capture the fact that unlike in a slow moving capital market, in a financial market attacks or withdrawals can be executed immediately. Thus, an investor may worry that even if he delays the withdrawal slightly, he may be unable to recover his original investment if other investors have already lined up to make redemptions.

We organize this section in two parts. First, we consider a stress test after the shock hits, i.e., only condition (2) holds, but not condition (1). As in our benchmark setup, we normalize the time at which the shock hits as 0. This part demonstrates how the discontinuous cost of waiting could severely limit the effectiveness of a disaster alert. Second, we consider a stochastic shock. The agents commonly know that the macroeconomic shock will hit the bank or the financial institution at some time in $[0, T]$. But they do not know or see when precisely the shock hits the bank or the financial institution, and how prepared the bank is to face the shock when it hits. They share the common belief $G(t)$ regarding when the shock may hit.

Skrzypacz (2018) for the co-determination of stress tests disclosure and capital requirements.
**Stress test after the shock hits**

The crucial difference from our benchmark setup is that if an agent attacks at time $t$, then his payoff is

$$u(t, \theta, N_t) = \begin{cases} u_0(t) & \text{if } r(t) = 0 \\ u_1(t) & \text{if } r(t) = 1 \end{cases},$$

where both $u_0(t)$ and $u_1(t)$ are Lipschitz continuous and decreasing in $t$, and for any $t$, $u_0(t) \geq u_1(t)$. As before, we normalize $u_0(0) = 1$. Thus, if at any time $t$, the agent waits for next $dt$ time, and the regime changes in the mean time, then the agent will get $u_1(t + dt)$ rather than $u_0(t + dt)$. Although $u_0(t)$ and $u_1(t)$ are continuous, if $u_1(t + dt)$ is strictly lower than $u_0(t)$ (as $dt \to 0$), then the cost of waiting is discontinuous.

As before, if the agents do not withdraw, then they get

$$v(t, \theta, N_T) = \begin{cases} g & \text{if } r(T) = 0 \\ l & \text{if } r(T) = 1 \end{cases}$$

(for simplicity we assume $g$ and $l$ are constant).

If $u_1(t) = u_0(t)$, then we have our benchmark setup. On the other extreme, if $u_1(t) = l$, then as soon as the regime changes, an agent who has not attacked already will get $l$ regardless of whether he attacks or not. In this case a disaster alert is “too late” in the sense that when the agents learn about the disaster, there is nothing they can do about it. In general, the disaster alert $\Gamma^\tau$ is valuable if $u_1(\tau) > l$. However, a disaster alert will have only limited success in dissuading the agents from attacking. To demonstrate this we specialize to a standard global game information structure (as mentioned in the model section).

**Proposition 1** Under the standard global information structure, and the disclosure policy $\Gamma^\tau$, the regime does not survive because of panic when $\theta \in \Theta^P(\Gamma^\tau) = [0, \hat{\theta}^\tau]$, where

$$\hat{\theta}^\tau = \frac{1}{1 + \frac{g-1}{1-u_1(\tau)}}.$$

Consider the case when there is no disaster alert. Then, an agent either attack immediately, or does not attack at all. It follows from standard global game argument (see Morris and Shin (2003)) that there is a unique equilibrium where the agents follow a cutoff strategy
– attack immediately if \( s_i < \hat{s} \), otherwise do not attack. Accordingly, the regime survives if and only if \( \theta \geq \hat{\theta} \), where
\[
\hat{\theta} = \frac{1}{1 + \frac{q-1}{1-\theta}}.
\]
Now suppose there is a disaster alert. If the agents follow the same strategy: attack if and only if \( s_i < \hat{s} \), then the alert will be triggered if and only if \( \theta < \hat{\theta} \). It follows from Lemma 1 that the agent who has not attacked, will attack if and only if the alert is triggered. If \( u_1(\tau) = l \), then after the alert is triggered, it is too late to act based on the alert, i.e., the agent gets the same if he attacks or not. Thus, from an ex-ante perspective, the strategy to not attack immediately, gives the same payoff regardless of whether there is a disaster alert or not. So, the same equilibrium remains, and disaster alert is completely ineffective. However, if \( u_1(\tau) > l \), then the disaster alert is valuable. This will incentivize the agents to wait for the alert. The higher the \( \lim_{\tau \to 0} u_1(\tau) \), the higher the incentive, and thus, the lower the threshold \( \hat{\theta}^{\tau} \). If there is no discontinuity (as in our benchmark model), i.e., \( u_1(\tau) \to u_0(0) = 1 \) as \( \tau \to 0 \), then \( \hat{\theta}^{\tau} \to 0 \). Thus, a timely disaster alert can eliminate panic.

If the regime may change anytime within the time window, it is possible that the agents can observe when the regime actually changes. Thus, the disaster alert may not be a deliberate policy, rather a natural feature. One can interpret the case in which the agents do not observe when the regime change as the static regime change game (See Inostroza and Pavan (2017)). If \( u_1(\tau) \) is close to \( u_0(\tau) \), then the fact that that agents can observe when the regime changes, could reduce the chance of panic. Note that the global game information structure does not satisfy the doubt assumption. It is easy to see that if the information structure satisfies the doubt assumption, then it can eliminate panic as long as \( u_1(\tau) \) is sufficiently close to \( u_0(\tau) \). However, if \( u_1(\tau) \) is not close to \( u_0(\tau) \), then the discontinuity of waiting cost severely limits the effectiveness of the fact that the agents learn when the regime changes, regardless of whether it is a deliberate policy or a natural feature. In the extreme if \( u_1(\tau) = l \), then agents will panic, regardless of whether they observe when the regime changes or not. Thus, a disaster alert policy is not only sub-optimal, but completely ineffective.

Next, we consider the case when the shock hits stochastically over time. We will consider the extreme case \( u_1(t) = l \), i.e., a post shock disaster alert is completely ineffective. However, a forward-looking stress test can send an early warning that the bank is not prepared to face the shock when it arrives. If the test is timely, then the chance that the shock
has already arrived is small. Therefore, the cost of waiting can be made negligible by conducting a timely stress test.

**Stress Test before the shock hits**

Nature chooses the bank’s preparedness $\theta$ at time 0, and the agents receive some noisy information about $\theta$ at time 0. As in our benchmark setup, we consider the same exogenous information structure that satisfies the doubt assumption. However, unlike in our benchmark model, the shock does not necessarily hit the bank or the financial institution at time 0. Suppose the shock hits at time $t_s$. We call the period $[0, t_s]$ the normal time, and $(t_s, T]$ the crisis time. The agents can make withdrawal at any time within the window $[0, T]$.\(^{16}\) For simplicity, we assume that the agents do not receive new information over time.\(^{17}\)

The principal does not know the private signals, and sets a timely disaster alert $I^\tau$. Note that at time $\tau$, it is possible that the shock has not arrived yet. Therefore, this disclosure requires due diligence. The principal checks if the bank is prepared to withstand the shock when it arrives. If $R(\theta, N_\tau) < 0$, then the bank will surely fail when the shock arrives (if it has not arrived already), and the disaster alert is triggered ($d^\tau = 1$). On the other hand, if $R(\theta, N_\tau) \geq 0$, then the bank will survive if the agents who have not withdrawn already, do not withdraw.

**Theorem 3** Under stochastically arriving shock, there exists $\tilde{\tau}$ such that a timely disaster alert $I^\tau$ with $\tau < \tilde{\tau}$ eliminates panic.

**Proof.** As in Lemma 1, the option value argument holds, i.e., an agent who has waited for the alert will not attack if the alert is not triggered. If an agent waits for the alert, then with probability $G(\tau)$ the shock hits the bank before the scheduled alert and with complementary probability it will hit later. Independent of whether the shock hits before or after the scheduled alert, the bank passes the stress test, i.e., $d^\tau = 0$ if $R(\theta, N_\tau) \geq 0$, and fails otherwise. If the bank fails the test, and the shock has already hit, then the investor will get $l$ if he attacks after the bank fails the test. However, if the bank fails the test, but

\(^{16}\)We will use the convention that at any time, first the nature decides whether the shock arrives or not, then the agents decide whether to withdraw or not (if not withdrawn already), then the principal decides whether to disclose any information.

\(^{17}\)It may be natural to assume that after the regime changes, the agents learn the news publicly. However, note that after learning this news, the agents get $l$ regardless of whether they withdraw or not. Therefore, this assumption does not make any difference.
the shock has not hit yet, i.e., the bank has not failed yet, then the investor will get $u_0(\tau)$ if he attacks after the bank fails the test.

Therefore, the expected payoff from playing $\mathcal{W}$ is

$$G(\tau) \left( \mathbb{P}(d^\tau = 0|s_i)g + \mathbb{P}(d^\tau = 1|s_i)l \right)$$

$$+ (1 - G(\tau)) \left( \mathbb{P}(d^\tau = 0|s_i)g + \mathbb{P}(d^\tau = 1|s_i)u_0(\tau + dt) \right).$$

$$\geq G(\tau)l + (1 - G(\tau)) \left( \varepsilon g + (1 - \varepsilon)u_0(\tau + dt) \right).$$

The inequality follows since $g \geq u_0(\tau + dt) \geq l$ and $\mathbb{P}(d^\tau = 0|s_i) > \varepsilon$ (Doubt). On the other hand, the payoff from playing $\mathcal{A}$ is $u(0) = 1$. As in theorem 1, using the Lipschitz continuity of $u_0$, we can say the expected net payoff from playing $\mathcal{W}$ as compared to $\mathcal{A}$ is at least

$$G(\tau)(l - 1) + (1 - G(\tau)) (\varepsilon(g - 1) - (1 - \varepsilon)K\tau)$$

So, $\mathcal{W}$ dominates $\mathcal{A}$ if

$$K\tau + \frac{G(\tau)}{1 - G(\tau)} \frac{1 - l}{1 - \varepsilon} < \frac{\varepsilon}{1 - \varepsilon} (g - 1).$$

Since $G(0) = 0$ and $G(.)$ has atomless density, the LHS in the above inequality converges to 0 when $\tau$ goes to 0, and it is continuously increasing in $\tau$. Therefore, for any given $\varepsilon > 0$, we can always find a $\tilde{\tau}$, such that $\mathcal{W}$ dominates $\mathcal{A}$ when $\tau < \tilde{\tau}$.

Thus, under a timely disaster alert, if $\theta \notin \Theta^L$, then since all the agents wait, the disaster alert is not trigger, regardless of whether the shock has already hit or not. This means $\Theta^P(\Gamma^\tau) = \emptyset$ when $\tau < \tilde{\tau}$. ■

5 Discussion

This paper considers a canonical regime change game where agents’ private signals could be arbitrarily correlated, and the principal does not have access to these private signals. She adopts a simple policy – a timely disaster alert, and she does not need ex-ante commitment to enforce such a policy. Yet, surprisingly, the policy completely eliminates panic. Even when new information arrives over time, a timely alert set for each time new information arrives stops the agents from panicking. We also saw how this insight can be used to design forward-looking stress tests. In reality, some of the features of our benchmark setup could
be different. In this section, we discuss the role of these assumptions, and evaluate whether they are essential or could be relaxed.

5.1 Sequential Rationality

We use sequential rationality to argue that an agent who has waited for the disaster alert will not attack after the alert is not triggered (option value lemma). If an agent believes that others are sequentially rational, then it follows from the option value lemma that he is not facing any strategic uncertainty after the alert. We leverage this to show that the agents will wait for the alert if it is set in a timely manner (timely alert lemma).

It is worth mentioning that in the epistemic game theory literature (See Dekel and Siniscalchi (2015) for a recent survey), the solution concept that uses the common knowledge of (sequential) rationality at the beginning of the game is called initial rationalizability. We are using this solution concept in an incomplete information setting. This problem has an intuitive connection to coordination with outside option example in the forward induction literature (See Pearce (1984), Kohlberg and Mertens (1986), Van Damme (1989), Ben-Porath and Dekel (1992)). The forward induction argument requires that agents continue believing in others’ sequential rationality even after seeing an unexpected history (See Battigalli and Siniscalchi (2002)). Given the fundamental uncertainty and the doubt assumption, it is always possible that the alert will not be triggered. Thus, neither the history $d^\tau = 0$ nor the history $d^\tau = 1$ is unexpected. Thus, any constraint on the beliefs conditional on unexpected histories, or the forward induction refinement is unnecessary for our result.$^{18}$

However, the readers may wonder that if a rational agent fears that others are not sequentially rational, then the strategic uncertainty after the alert remains. Therefore, the agents may not want to wait for such alert, and this may lead to panic. Suppose that there is a chance $\eta$ that even after the disaster alert is not triggered, some agents who have waited will behave irrationally, and attack, causing a regime change. The following proposition shows that if the probability $\eta$ is sufficiently small, a timely disaster alert will eliminate panic.

$^{18}$Also, unlike the coordination with outside option example, in our setup all the agents can attack early. When both agents have outside options (or attacking early in our setup), they face another coordination problem - whether to take the outside option, or play the coordination game that follows.
Proposition 2 Suppose that even after the alert is not triggered, because of irrational attacks, the chance of regime change is \( \eta \). If \( \eta < \frac{g-1}{g-l} \), then there exists a \( \hat{\tau}^\eta > 0 \) such that \( \Gamma^\tau \) with \( \tau < \hat{\tau}^\eta \) eliminates panic.

Recall that while the cost of waiting for a timely alert is almost zero, there is a strictly positive benefit from waiting. If the agents are not certain about the sequential rationality of the other agents, then this benefit is lower. However, if \( \eta \) is small enough, then the benefit remains strictly positive. In this sense the result is robust even if we relax the assumption of common knowledge of (sequential) rationality.

5.2 (Un)necessary Doubt

The policy of a timely disaster alert works with a relatively flexible set of exogenous information structure. The only restriction we impose on the exogenous information is the doubt assumption. Note that this assumption is stronger than saying that there is an upper dominance region. It says that regardless of the private signal an agent believes that \( \theta \) could be in the upper dominance region. This assumption can be weakened depending on the heterogeneity of agents’ beliefs as the following example shows.

Example 1 The regime change function \( R(\theta, N) = \theta - N \). Nature draws \( \theta \) from \( U[-1, 2] \), and agents receive independent private signals \( s_i \in \{l, m, h\} \) according to the following conditional distribution \((p \in (0, 1))\)

\[
\begin{array}{ccc}
\theta \in \Theta^L = [-1, 0) & l & m & h \\
\theta \in [0, 1) & p & (1-p) & 0 \\
\theta \in \Theta^U = [1, 2] & \frac{1}{2}(1-p) & p & \frac{1}{2}(1-p) \\
\end{array}
\]

In the above example 1, \( f(s_i|\theta) \) does not have full support. If an agent receives the signal \( l \), then the agent knows for sure that the fundamental of the regime is not in the upper dominance region, i.e., \( \theta < 1 \). Hence, the doubt assumption is violated.

In this case, agent receiving signal \( l \) knows that attacking immediately is not a mistake if others also attack immediately. Suppose that all agents receive the public signal \( s = l \). Then, even under any timely disclosure policy \( \Gamma^\tau \), a possible equilibrium outcome is that all the agents attack immediately. This shows that under the general information structure, assumption 1 is indeed a necessary condition for our main result.
However, this does not mean that a given information structure must satisfy assumption 1 for the result to be true. Let us go back to example 1, but now suppose that the signals are not public. In particular, let us suppose that signals are conditionally independent.

First, note that the agents who receive $s_i = m$ or $h$ believes that there is positive chance that the fundamental of the regime is in the upper dominance region, i.e., \( P(\theta \geq 1|s_i = h) = \frac{2p}{1+p} > P(\theta \geq 1|s_i = m) = \frac{1-p}{2-p} \). Hence, the doubt assumption holds for $s_i = h, m$ for any $\varepsilon < \frac{1-p}{2-p}$, and agents receiving these two signals will not attack immediately (Lemma 2) under an appropriate timely disaster alert.

Now consider the agent who receives signal $s_i = l$. He knows that the fundamental is not in the upper dominance region, and thus if all other agents attack immediately, the alert will be triggered for sure. However, since signals are not public, he understands that some other agents may have received signal $s_{-i} = m$ or $h$, and thus, not attacking. Only agents who have received $s_{-i} = l$ may attack before the time of disclosure, and thus, the maximum attack is the share of agents with signal $l$. This means that the alert cannot be triggered if the realized fundamental $\theta$ greater than the fraction of agents with signal $l$, i.e., \( P(d^\tau = 0|l) \) is at least

\[
\mathbb{P}(\theta \geq \mathbb{P}(s_{-i} = l)|\theta \in [0, 1])\mathbb{P}(\theta \in [0, 1]|l) = P(\theta \geq \frac{1}{2}(1-p)|\theta \in [0, 1]) \times \frac{1-p}{1+p} = \frac{1-p}{2}.
\]

This shows that even the agent who receives $s_i = l$ believes that there is a strictly positive probability that the disaster alert will not be triggered. That means attacking immediately (strategy $A$) might be a mistake and thus, when the alert is set in a timely manner, the strategy of wait and see ($W$) is the dominant one. \(^{19}\)

Now suppose that the information structure is as follows: with probability $\alpha$, $s_j = s_i$ and with probability $(1 - \alpha)$ the $s_j$ is a conditionally independent signal. If $\alpha = 1$, then this captures the public information case, and if $\alpha = 0$, this captures the conditionally independent signal case. Lower $\alpha$ means more heterogenous beliefs. For any given $\alpha < 1$,

\[
\mathbb{P}(d^\tau = 0|l) \geq (1 - \alpha)\frac{1-p}{2}.
\]

This shows that if $\alpha$ is away from 1, i.e., there is enough heterogeneity in agents’ beliefs,

\(^{19}\)Note that this argument requires that the agents with signal $l$ not only believes that others are rational, but also that the agents with signal $m$ and $h$ believe that others are rational.
the principal can eliminate panic. In this sense, the doubt assumption is unnecessary.

5.3 *Costly Delay*

Consider the capital outflow example. However, unlike in our benchmark setup, suppose the investors receive flow payoff from staying. If an investor has not exited by time $t \in [0, T]$, he receives a flow payoff at time $t$ depending on the underlying fundamental $\theta$ and aggregate exit so far $N_t$ as follows.

$$
\tilde{r}(\theta, N_t) = \begin{cases} 
\tau & \text{if } R(\theta, N_t) \geq 0 \\
r & \text{if } R(\theta, N_t) < 0.
\end{cases}
$$

If $R(\theta, N_T) \geq 0$ (the regime survives), then the investors who do not exit will earn a flow payoff $\tau$ forever. But if $R(\theta, N_t)$ becomes less than 0 at some $t$, then the investor who does not exit will start getting a flow payoff $r$ from time $t$ onwards. Thus, if the investor stays, then his payoff is

$$
v(\theta, (N_t)) = \int_{t=0}^{T} e^{-\beta t}\tilde{r}(\theta, N_t)dt + \int_{T}^{\infty} e^{-\beta t}\tilde{r}(\theta, N_T)dt,
$$

where $\beta > 0$ is the discount rate. On the other hand, if an investor exits at some time $t$, then he switches to a safe investment project, which yields a fixed flow return of $r > 0$, where

$$
\tau > r > r.
$$

Note that unlike in the benchmark setup, the regime can change at any time $t$, rather than only at $T$. The flow payoff acts as endogenous disaster alert – whenever the flow payoff from keeping the investment at the emerging market becomes $r$, the agents learn that the regime has changed. Unlike in our benchmark setup, delayed attack may not be costly since the investor could earn $\tau$ rather than $r$ by exiting later.

Government in the emerging economies often imposes tax on capital flight. Clearly a tax would discourage the investors from exiting the market. However, this could discourage the investors to invest in the first place (See Mathevet and Steiner (2013)). We make a different argument. A tax on capital flight makes waiting costly, i.e., if an investor exits then he should exit earlier than later. Suppose that the investor has to pay a tax at a rate $\mu \in (0, 1)$ on the flow payoff the investor has earned until time $t$. Thus, his payoff from
withdrawing at time $t_i \in [0, T]$ is

$$u(\theta, t_i, (N_t)) \equiv \int_{t=0}^{t_i} e^{-\beta t} (1 - \mu) \bar{r}(\theta, N_t) dt + \int_{t_i}^{\infty} e^{-\beta t} r dt.$$ 

**Assumption 2** (1) $\mu > 1 - \frac{r}{\bar{r}}$ and (2) $T < \frac{1}{\beta} \ln(1 + \frac{r - \bar{r}}{\bar{r}})$.

The first restriction in Assumption 2 ensures that the tax rate is high enough such that $(1 - \mu)\bar{r} < r$. This implies that $u$ is decreasing in $t$, i.e., if an investor exits, he should exit as early as possible. The second restriction in Assumption 2 ensures that the time window is sufficiently small. Otherwise, an investor may accumulate significant flow payoff over time and may not want to exit because of the significant exit tax.

**Proposition 3** In the capital outflow game, under Assumption 2, the investors do not panic. That is, $\Theta^P = \emptyset$.

It follows from the second part in Assumption 2 that after seeing $\bar{r}(\theta, N_t) = \bar{r}$, an investor will exit right the next instance, and it follows from the first part in Assumption 2 that delayed exit is costly. Therefore, any agent who did not exit early would only exit later when $\bar{r}(\theta, N_t) = \bar{r}$. In other words, the option value argument holds here. Since, the disaster alert is continuously in pace, it follows from Theorem 1 that all the agents who believe that the flow payoff in future can be $\bar{r}$ with positive probability regardless of what others do (Assumption 1) would never exit unless $\bar{r}(\theta, N_t) = \bar{r}$ is realized. This completely eliminates the panic.

### 5.4 Reversibility and Panic

One may think that if attack is a reversible action, then it will make the result even stronger. After all, if the agent who had left can come back, it will reassure the agents who stayed. However, if attack is a reversible, an agent can take the following strategy $(W^C)$: attack right away, and reverse the action only if the alert is not triggered. This could trigger the alarm although it was not warranted. In the following example we consider a regime change game in which both actions are reversible, and the payoff depends on the duration of the actions.

**Example 2** The investors decide whether to stay in (not attack) or stay out (attack). The payoffs depends on the duration of each action. If he stays in for a duration of $t$, then he
gets a flow return $r$ for the duration that he stays out ($T - t$), and a higher return ($g$) or lower return ($l$), depending on the fate of the regime, weighted by the duration that he stays in. The regime change function $R(\theta, N) = \theta - N$. Thus, the investor’s payoff is

$$r(T - t) + t \cdot (g 1(\theta \geq N_T) + l 1(\theta < N_T)),$$

where $g > r > l$.

The strategy $W^c$ gives expected payoff $\mathbb{P}(d^\tau = 1|s_i)rT + \mathbb{P}(d^\tau = 0|s_i)(r\tau + (T - \tau)g)$. On the other hand, the strategy $W$ gives expected payoff $\mathbb{P}(d^\tau = 1|s_i)((T - \tau)r + \tau l) + \mathbb{P}(d^\tau = 0|s_i)(T g)$. Therefore, the net payoff from playing $W^c$ as opposed to $W$ is

$$\mathbb{P}(d^\tau = 1|s_i)(\tau(r - l)) + \mathbb{P}(d^\tau = 0|s_i)(\tau(r - g)).$$

Therefore, if an agent believes that the alert is sufficiently likely to be triggered, he will play $W^c$ rather than $W$.

When attacking is reversible, agents (especially with low signals) may decide to stay out unless the alert says otherwise. This strategy can trigger the alert (especially when $\theta$ is low). To demonstrate this, we specialize to a standard global game information structure.

**Proposition 4** Under the standard global game information structure, when attack is reversible, and the payoffs are as specified in example 2, there exists an equilibrium in which the agents with $s_i < \hat{s}$ plays $W^c$, and the ones with $s_i \geq \hat{s}$ plays $W$. Accordingly, for $\theta \in (0, 1 - \frac{\hat{s}}{\hat{s} + r})$ the regime changes although it is not warranted.

**References**


Bernanke, Ben S. 2013. “Stress Testing Banks: What Have We Learned?”


Appendix

Proof of Theorem 2 For $m = 0, 1, 2, \ldots, M - 1$, let $\mathcal{W}^{m+1}$ be the strategy in which an agent wait until the time $(t_m + \tau)$ for the $(m + 1)$ th disaster alert and follow all the recommendations along the path, i.e., attack only when $d_{m'}^{m'+\tau} = 1$ for all $m' < m$, but not the next disaster alert, and attack at time $t_{m+1}$. Under the strategy $\mathcal{W}^{M+1}$, agents wait for all disaster alerts and follow all recommendations.

Lemma 3 Under the extended disclosure policy $\Gamma^\tau$, for any noisy signal $\{s_i^m\}_{m=0}^M$, the only rationalizable strategies are $A$ and $\{\mathcal{W}^{m+1}\}_{m=0}^M$.

Proof. Because of the delay cost, attacking at any time when there is no new arrival of information is strictly dominated by attacking at an earlier time with the same information. Hence, attack can only happen at time $t_m$ or $t_m + \tau + dt$ for $m = 0, 1, \ldots, M$. The agents who have waited for the $(m + 1)$ th disaster alert will attack right away if $d_{m}^{m+\tau} = 1$. It follows from the option value argument that if $d_{m}^{m+\tau} = 0$, regardless of $\{s_i^{m'}\}_{m'\leq m}$, an agent would not attack. Otherwise, the new information disclosed at $t_m + \tau$ does not have any value and the agent should attack earlier without waiting for the new information. Hence, each rationalizable strategy can be described as the waiting until the $(m + 1)$ th disaster alert for $m = 0, 1, 2, \ldots, M$, or not wait for any disaster alert at all and attack right away $A$ (or equivalently $\mathcal{W}^0$).

Lemma 4 Given generalized doubt assumption, under the extended disclosure policy $\Gamma^\tau$, where $\tau < \hat{\tau}$, for any signal realization $\{s_i^m\}_{m=0}^M$, the only rationalizable strategy for an agent is $\mathcal{W}^{M+1}$, i.e., wait for all disaster alert and follow the principal’s recommendation.

Proof. Under any rationalizable strategy, if any disaster alert is triggered before time $t_M$, all agents attack before $t_M$. Now, suppose the disaster alerts have not been triggered before time $t_M$. For the agents who have waited until the $M$th disaster alert, $\mathcal{W}^{M+1}$ strictly dominates $\mathcal{W}^M$ (same argument as in Lemma 2). Hence, regardless of which rationalizable strategy agents take, if the previous alert was not triggered ($d_{M-1}^{M-1+\tau} = 0$), there is no attack between the $M$th alert and the $(M + 1)$th alert, and hence, $d_{M}^{M+\tau} = 0$. Otherwise, if $d_{M-1}^{M-1+\tau} = 1$, then $d_{M}^{M+\tau} = 1$. Therefore, under timely alert, $d_{M}^{M+\tau} = d_{M-1}^{M-1+\tau}$. This means that if the $M$th alert is not triggered, then attacking between $M$ th alert and the $(M + 1)$ th alert is not rational. For that reason, the problem at time $t_{M-1}$ is exactly the same as the one at time $t_M$. Repeating the same argument, $\mathcal{W}^{M+1}$ strictly dominates $\mathcal{W}^{M-1}$ when
and accordingly \( d^{M+\tau} = d^{M-1+\tau} = d^{M-2+\tau} \). Repeating the same argument again, it is easy to prove that \( W^{M+1} \) strictly dominates \( W^m(m = 1, 2, \ldots, M) \) and \( A \). □

Hence, under the extended disclosure policy \( I^{\tau} \) with \( \tau < \hat{\tau} \), regardless of their signals, under the unique rationalizable strategy \( W^{M+1} \), agents never attack a regime with \( \theta \notin \Theta^L \). Following the same argument as in the proof of Theorem 1, \( \Theta^P(I^{\tau}) = \emptyset \). □

**Proof of Proposition 1** The regime survives only when the fundamental \( \theta \) is no lower than the mass of agents who choose \( A \), or \( W \). The payoff for taking \( A \) is 1, while the payoff for taking \( W \) is \( g \) when \( d^\tau = 1 \) (or equivalently \( \theta \geq N_0 \)) and \( u_1(\tau) \) when \( d^\tau = 0 \) (or equivalently \( \theta < N_0 \)).

For agent \( i \) who has received private information \( s_i < -\frac{1}{2}\sigma \), he knows that \( \theta < 0 \) and thus \( d^\tau = 1 \) for sure and thus he will take \( A \). Hence, the dominance region of \( A \) is \([\theta - \frac{1}{2}\sigma, -\frac{1}{2}\sigma]\). Similarly, the dominance region of \( W \) is \([1 + \frac{1}{2}\sigma, \bar{\theta} + \frac{1}{2}\sigma]\).

Consider an agent with signal \( s_i \). If he believes that

\[
P(d^\tau = 0|s_i) \geq \frac{1}{1 + \frac{g-1}{1-u_1(\tau)}},
\]

then he will play \( W \) rather than \( A \). Following the iterated elimination arguments as in global games, we can have

\[
\hat{s}^\tau = \hat{\theta}^\tau + \sigma F^{-1}(\hat{\theta}^\tau) \quad \text{and} \quad \hat{\theta}^\tau = \frac{1}{1 + \frac{g-1}{1-u_1(\tau)}}. \quad \square
\]

**Proof of Proposition 2** Consider an agent with signal \( s_i \). The net payoff from playing \( W \) as compared to \( A \), \( D(I^{\tau}, s_i) \), is

\[
\mathbb{P}(d^\tau = 1|s_i)(u(\tau) - u(0)) + \mathbb{P}(d^\tau = 0|s_i)(\mathbb{E}[(1 - \eta)g(\theta, N_T) + \eta l(\theta, N_T)|s_i] - 1).
\]

Using Lipschitz continuity of \( u(t) \) and Assumption 1, we have

\[
D(I^{\tau}, s_i) \geq -(1 - \varepsilon)K\tau + \varepsilon((1 - \eta)(g - 1) + \eta(l - 1)).
\]
If $\eta < \frac{g-1}{g-1}$, then a timely disaster alert policy $\Gamma^\tau$ where $\tau < \hat{\tau}^\eta \equiv \frac{\epsilon}{1-\epsilon} (1-\eta)(g-1)_{\gamma(1-I)}$ eliminates panic. □

Proof of Proposition 3  Suppose that the flow payoff switches from $\bar{r}$ to $\underline{r}$ at time $\tau$. Then, the expected payoff from waiting and not exiting at $\tau + dt$ is

$$v(\theta, (N_t)_{t<\tau}) = \int_{t=0}^{\tau} e^{-\beta t} \bar{r}(\theta, N_t) dt + \int_{t=\tau}^{\infty} e^{-\beta t} r dt,$$

while the expected payoff from exiting right away is

$$u(\tau, \theta, N_\tau) = \int_{t=0}^{\tau} e^{-\beta t} (1-\mu) \bar{r}(\theta, N_t) dt + \int_{t=\tau}^{\infty} e^{-\beta t} r dt.$$

So, the net payoff from waiting as compared to exiting right away is

$$\int_{0}^{\tau} e^{-\beta t} \mu \bar{r}(\theta, N_t) dt - e^{-\beta \tau} \frac{\bar{r} - \underline{r}}{\beta} \leq \frac{1}{\beta} \left[ \mu \bar{r} - e^{-\beta \tau}(\mu \bar{r} + r - \underline{r}) \right].$$

Note that the above inequality holds true for any possible $\bar{r}$ before time $\tau$ (since $\bar{r} \leq \bar{r}$).

Based on the above inequality, one can easily check that, for any possible tax rate $\mu \in (0,1)$, as long as $\tau < \frac{1}{\beta} \ln \frac{\bar{r} - \mu + \underline{r}}{\mu}$, $v(\theta, (N_t)_{t<\tau}) < u(\tau, \theta, N_\tau)$ and thus investors would strictly prefer to exit immediately. Hence, if the second part of Assumption 2 holds true, i.e., $T < \frac{1}{\beta} \ln \frac{\bar{r} - \mu + \underline{r}}{\mu}$, independent of the history $(N_t)$ and the tax rate $\mu$, investors would immediately exit once observing $\bar{r} = r$ at any time within the time window $[0, T]$.

It follows from the first part of Assumption 2 that $u(\theta, t, N)$ is decreasing in $t$. Therefore, the option value argument holds true (Lemma 1). Let us define $\mathcal{T} = \{ t \in [0, T] | r(\theta, N_t) = r \}$. If $\mathcal{T} = \emptyset$, then the switch does not happen, otherwise, let us write the first time when the flow payoff switches to $r$ as $t_r := \min \mathcal{T}$. With the option value argument, the only rationalizable strategies are (1) $\mathcal{A}$: exiting at time 0; and (2) $\mathcal{W}$: waiting and only exiting at $t_r + dt$ (not exiting if $\mathcal{T} = \emptyset$). Under these two strategies, the switch can only happen at time $0 + dt$. With the doubt assumption, the expected payoff from $\mathcal{A}$ is $\frac{\epsilon}{\beta}$, while that from $\mathcal{W}$ is

$$\epsilon(\bar{r}) + (1-\epsilon) \left( \frac{1- e^{-\beta dt}}{\beta} \bar{r} + \frac{e^{-\beta dt}}{\beta} r \right).$$

Obviously, $\mathcal{W}$ strictly dominates $\mathcal{A}$. Hence, if $\theta \notin \Theta^L$, the investors will not panic and start exiting. □
Proof of Proposition 4  Suppose the agent play such a cutoff strategy. Then, for any $\theta$, the aggregate attack at time 0 is $F((\hat{s} - \theta)/\sigma)$. Clearly this is decreasing in $\theta$. Therefore, there is a $\hat{\theta}$ such that the alert is triggered ($d^r = 1$) if and only if $\theta < \hat{\theta}$, where $F((\hat{s} - \hat{\theta})/\sigma) = \hat{\theta}$. An agent with private signal $s_i$ believes that the net expected payoff from playing $W^c$ compared to $W$ is

$$\left(1 - F\left(\frac{s_i - \hat{\theta}}{\sigma}\right)\right)(\tau(r - l)) + F\left(\frac{s_i - \hat{\theta}}{\sigma}\right)(\tau(r - g)).$$

Clearly this is decreasing in $s_i$. Consider the marginal agent with signal $s_i = \hat{s}$. He must be indifferent between playing $W^c$ and $W$. Substituting $F((\hat{s} - \hat{\theta})/\sigma) = \hat{\theta}$, we get that the net benefit of the marginal agent is

$$\tau(r - l) - \hat{\theta}\tau((g - r) + (r - l)) = 0 \Rightarrow \hat{\theta} = \frac{1}{1 + \frac{g - r}{r - l}}.$$

Thus, $\hat{s} = \hat{\theta} + \frac{1}{\sigma}F^{-1}(\hat{\theta})$ constitutes an equilibrium, and in this equilibrium a regime with fundamental below $\hat{\theta}$ but above 0 will not survive because of panic-based runs. □