Systemic Bank Panics in Financial Networks*

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Abstract

By incorporating financial panic into the analysis, this paper studies how financial stability depends on the interconnectedness of the interbank network and the information and beliefs held by market participants. Financial connections between banks not only transmit fundamental shocks, but also propagate and intensify the panic in financial markets. A novel mechanism is found to show that, when no bank falls into distress before its creditors make their withdrawal decisions, financial networks with a more diversified pattern of interbank liabilities will trigger more panics and will thus be more fragile. When one bank in the network receives a shock that is large enough to make it insolvent, creditors may be uncertain about the financial linkages between their bank and the initial distressed bank. When such a crisis is underway, I show that information disclosure is likely to trigger more panics from the initial distressed bank and facilitate financial contagion. Relevant regulations and policies are also discussed in this paper.

JEL Classification Numbers: D82, D85, G01, G33, L14

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"...more-numerous and more-complex linkages also appear to make it more difficult for institutions to address certain types of externalities, such as those arising from incomplete information or a lack of coordination among market participants. These externalities may do little harm or may even be irrelevant in normal times, but they can be devastating during a crisis..."

——Janet Yellen, American Finance Association Joint Luncheon (January, 2013)

Introduction

The panic among market participants, in response to shocks and/or due to a lack of information, lies at the nexus of the Great Recession. Panics occur when financial market participants become concerned about the underlying quality of assets or the precautionary actions taken by other players in the market. The financial panic forces market participants to make withdrawals from banks, redeem investments from Money Market Funds and refuse to renew loans. During the great recession, in addition to traditional bank runs, panics occurred in the repurchase or "repo" market (Gorton and Metrick, 2012), the asset-backed commercial paper (ABCP) markets (Covitz et al., 2010), and the auction rate securities (ARS) market (Han and Dan, 2011). As a result, financial markets experienced significant increases in haircuts and interest rates, costly liquidations, and fire sales, which led to further declines in asset prices and contributed to the market collapse.

In this paper, I focus on bank panics and examine the stability of interbank networks from the perspective of panic-driven runs. The collapse of the Reserve Primary Fund in 2008 illustrates how financial linkages can facilitate the spread of bank panic from one institution to others. The bankruptcy declaration of Lehman Brothers on September 15, 2008 triggered a panic in the financial market. The Reserve Primary Fund, which held \$785 million in commercial papers issued by Lehman Brothers, was forced to pay out \$10.8 billion in redemptions and faced about \$28 billion worth of additional withdrawal requests. The fund's sponsor did not have sufficient liquid assets to fulfill all redemptions and the fund's net asset value fell below \$1 per share, or "broke the buck." The panic spread into the entire money market funds sector, resulting in a credit market freeze. The demise of Northern Rock in 2007 in the United Kingdom was also mainly caused by panic amongst wholesale depositors.[1] As Janet Yellen noted in her speech at the 2013 American Finance

[1]See Shin (2008) for a detailed description of Northern Rock's crisis.

Association luncheon, the incomplete information and lack of coordination among creditors could be key ingredients of the mechanism for financial market crashes, especially when financial institutions are interconnected in a complex network.

Bank panic is a systematically important factor affecting financial stability. As defined by Calomiris and Gorton (1991), a (systemic) bank panic occurs when bank debt holders at all or many banks in the banking system suddenly demand that banks convert their debt claims into cash to such an extent that the banks suspend convertibility of their debts into cash. The interconnected nature of financial networks allows panics to be contagious. The liquidity risk of one bank, originating from panics among its creditors and/or adverse shocks, exacerbates the counterparty risk faced by the other banks, thereby facilitating panics among their creditors, and contributing to the financial contagion. Hence, the contagious nature of panics in financial networks will not only have an adverse impact on each individual bank's liquidity, but will also have a systemic impact that could potentially bring all or many banks into distress.

This paper makes the first attempt to incorporate panic-based runs in financial networks into the analysis of the stability of financial networks. An interbank network is more *fragile* (or less *stable*) if each bank in the network is more likely to default. I model the panic among financial market participants as a self-fulfilling prophecy. Thus, panics can be very sensitive to the information and expectations held by market participants about the financial health of institutions as well as the actions that will be taken by others.

Introducing panics into the analysis enables us to better understand how financial fragility is influenced by the pattern of financial linkages. It also helps us to understand the impact of information disclosure and creditors' beliefs on financial stability. From the perspective of panic-based runs, this paper sets out to address the following questions: How does the network structure affect financial fragility? Does the position of a bank in the network affect its soundness? What is the impact of information disclosure on the fragility of financial networks?

I construct a stylized model of *financial networks* formed by interbank liabilities. The exogenously given network structure specifies the creditor banks for each bank, and the face value of the interbank loans that banks need to repay to each of their creditor banks. Every bank has its own continuum of creditors (or depositors). All (non-bank) liquidity suppliers, including retail depositors, wholesale depositors, and short-term creditors are called *creditors* in this model.

The model captures the maturity mismatch in banks' assets and liabilities. Before long-term investments mature, each bank will use its liquid assets, as well as the interbank repayments it has received, to repay its interbank loans and meet withdrawals from its creditors. Every bank experiences some liquidity shock to the value of its liquid assets, and creditors receive noisy private information about their bank's liquidity shock. This information helps creditors to learn about the realization of liquidity shock (fundamental uncertainty), as well as the beliefs and strategic withdrawal decisions of other creditors connected to this bank (strategic uncertainty). When banks are interconnected, the creditors of one bank are also apprehensive about its counterparty risk, or what proportion of the interbank liabilities its debtor bank will be able to repay. Hence, creditors' withdrawal decisions depend upon how the network structures transmit and propagate these counterparty risks.

I first consider the case where each bank in the network faces a fixed distribution of liquidity shocks, but no bank is forced to default before outside creditors have made their strategic withdrawal decisions. Creditors' withdrawal decisions will determine the solvency of banks ex-post. This scenario will be referred to as *normal times*. I restrict my attention to *symmetric and regular networks*, where each bank's pattern of financial linkages is identical and the total claims and liabilities of all banks are equal.

A financial network is said to be more *diversified* if each bank makes its financial linkages less dense either by connecting to more counterparties or by distributing its interbank liabilities more evenly across a fixed number of counterparties. From the perspective of panic-based bank runs, I investigate whether more diversified patterns of interbank liabilities could help to make the financial system more stable.

I find a novel mechanism to show that financial networks with more diversified connections will trigger more panics and thus make the system more fragile. When banks build more diversified financial connections, the distribution of the total interbank repayments becomes less spread (more centered). For a given distribution of liquidity shocks to each bank, the shift in the distribution of interbank repayments essentially reduces the probability that a bank will have a very low (or high) capability of meeting its obligations and concurrently increases the probability that the bank will have an intermediate ability to meet its liabilities. As long as the bank can successfully fulfill its obligations, no matter how strong its capability is, the incentive for creditors to resist running on the bank is fixed (i.e., the interest rate is fixed). By contrast, in the default regime, as discussed in Goldstein and Pauzner (2005), creditors have fewer ncentives to run if a bank's capability of rolling over withdrawals is lower (because of the lower recovery rate). Hence, the shift in the distribution of the bank's capability to meet its obligations could provide extra incentive for creditors to run, thus making the system more fragile.

During normal times, I find that a symmetric and regular interbank network is more fragile either when the liquidity shocks to banks are more correlated or when banks have higher exposures (captured by the total interbank lending) to other banks. Based on these findings, the provision on "single counterparty exposure limits" in the Dodd-Frank Act (Section 165(e)), which attempts to prevent one institution's problems from spreading to the rest of the system by limiting each financial institution's exposure to any single counterparty, could be effective in promoting financial stability by restricting the aggregate exposure of each bank. However, it also provides incentives for financial institutions to build less dense and more diversified linkages, which could endogenously create more panics and undermine financial stability.

Empirical studies suggest that the interbank market has a core-periphery structure, in which large money center banks (the core) have links with each other and with a large number of peripheral banks, whereas the banks in the periphery have links with just a few banks in the core.[2] In this paper, I also investigate the financial fragility of *core periphery networks*. The core banks in these networks act as intermediaries for other (periphery) banks with fewer connections and lower exposures. I find that the core banks with more counterparties and higher total interbank lending will be more prone to panic-based runs than the periphery banks. I also show that that systematic risk increases with the size of the core banks and the volume of their interbank lending.

I next consider the case where one bank in the network receives a shock that is large enough to make it insolvent and default on all of its interbank loans, independent of creditors' reactions. The bank will become distressed even if all creditors stay with the bank. Other banks are exposed to the standard liquidity shocks as in the baseline model. As in Caballero and Simsek (2013), due to the complexity of the network, they may not have perfect information about how their bank is linked to the distressed bank, even if they know the structure of network and/or the identity of the distressed bank. Without information about the financial linkages, creditors may hold the *neutral* belief that each bank in the network has the same probability of being the distressed bank. Alternatively, they could act as *max-min* agents and hold the *cautious* belief that the initial distressed bank is the

[2] See Bech and Atalay (2010), van Lelyveld et al. (2014), Craig and Von Peter (2014) for empirical evidence for interbank markets in the U.S. and other countries.

largest counterparty of their bank, as in Caballero and Simsek (2013). In this context, I investigate the effects of information disclosure on endogenously generated panics and the extent of financial contagion.

I show that, in a *complete network*, i.e., a network in which each bank is connected to every other bank, financial contagion is independent of the creditors' beliefs or information about the location of the "bad apple" in the system. Hence, when the financial network is sufficiently diversified, it is not worthwhile to conduct a costly investigation (e.g., a stress test), to make this information available.

By contrast, in a *ring* or *circle* network, where each bank borrows from and makes loans to only one bank in the system, the creditors of the neighboring bank, which is the sole creditor bank of the distressed bank, will run aggressively if they understand how their bank is connected to the distressed one. This is because they understand that the sole counterparty of their bank will default on all interbank liabilities. The negative impact of the panic on one bank will be transmitted to its creditor banks since it increases their counterparty risk, thereby triggering more panics. Hence, the panic originating from the sole creditor bank of the distressed bank will facilitate financial contagion and have a destabilizing effect on the financial system.

In contrast to Alvarez and Barlevy (2014), who found that voluntary or mandatory information disclosure could reduce the uncertainty of outside investors and make the financial system more robust, I find that, from the perspective of panic-based runs, if the size of undiversified network is relatively small, each bank in the system will be more fragile under information disclosure. By a similar argument, I show that a less diversified network of relatively small size could be more fragile under complete information (or information disclosure).

In the absence of any information about the distressed bank's location, the creditors' beliefs will play an important role in propagating financial contagion. The financial fragility of less diversified networks will be more sensitive to creditors' beliefs about the distressed bank. The ring network, being the least diversified, is the most fragile if creditors are cautious and believe that the sole counterparty is the distressed bank. By contrast, the ring network is the most robust one when creditors hold neutral beliefs and assign the same probability for each bank being the distressed one. Under the assumption that creditors are *max-min* agents who assume the worst, as in Caballero and Simsek (2013), information disclosure could be welfare improving as it reduces the ambiguity in the system.

The panic among market participants is an important source of financial instability and it can be contagious when banks are interconnected in a network. Adopting the global game approach, I model panics as an equilibrating phenomenon and show that the extent of panics is crucial in determining the fragility of financial networks.

The conventional wisdom about financial networks - "robust yet fragile" - is that the interconnectedness of banks is robust during normal times, but fragile during bad times. I take a step further and examine the financial fragility of different network structures from the viewpoint of endogenous panics. I show that less diversified networks are more robust during normal times, but could be very sensitive to creditors' information and beliefs about the exact linkage of the distressed bank to their bank when a crisis is underway. Moreover, I find that information disclosure could have a destabilizing effect on the financial market because it triggers contagious panics and facilitates financial contagion.

Literature Review

This paper is mainly related to three strands of literature: (1) the relation between systemic risk and network structures, (2) the role of information in financial fragility, and (3) the global game models on panic-based bank runs and crises.

Following the seminal work of Allen and Gale (2000) and Freixas et al. (2000), a growing body of literature argues that certain financial network structures make the financial network more fragile to exogenous shocks. The existing literature investigates the relation between financial stability and network structure from the perspective of direct contractual linkages, such as interbank liabilities (e.g., Acemoglu et al. (2015b)), cross holding of deposits (e.g., Babus (2015)) or equities (e.g., Elliott et al. (2014)), or from the fire sales and other pecuniary externalities (e.g., Zawadowski (2013)).For more examples, see Nier et al. (2007), Gai et al. (2011), Greenwood et al. (2015), Glasserman and Young (2015), and surveys by Allen and Babus (2009) and Cabrales et al. (2015). My baseline model is based on the model of Eisenberg and Noe (2001) in which financial institutions are linked via unsecured interbank liabilities. In contrast to the literature on financial stability and network structure, I examine the fragility of financial networks with a focus on endogenously generated panics among market participants. From this perspective of panic-based runs, I provide new insight into how financial fragility depends on the network structure. For example, Acemoglu et al. (2015b) argue that more diversified interbank networks are good at absorbing small exogenous shocks, and also good at transmitting large shocks. This paper shows that more diversified patterns of financial linkages are more fragile during normal times, but could be more robust when creditors have perfect information about the financial linkages when a crisis is underway.

In my model, the network structure is assumed to be endogenously given. The literature on network formation has generally overlooked the liquidity risk endogenously generated by the coordination game among short-term creditors or depositors. For example, Babus (2015) studies the network formation problem and its implications for financial stability when banks face exogenous withdrawal shocks. For more examples, see Lagunoff and Schreft (2001), Castiglionesi and Navarro (2008), Zawadowski (2013),Cabrales et al. (2014), Di Maggio and Tahbaz-Salehi (2014), Farboodi (2014), e Castro (2015) and Wang (2015). In this paper, I show that the endogenous panic-based run in financial markets is a crucial factor in understanding the systemic risk. I show that heavier connections between financial institutions not only increase the risk of default for the corresponding banks, but also make the financial system more fragile. Over-diversification of connections could have similar effects during normal times. Hence, the externality induced by financial linkages in transmitting the panic among one bank's creditors into other banks should be incorporated into the analysis of optimal network formation.

I also investigate the role of information in financial contagion when there is a distressed bank in the financial network ex-ante independent of market participants' reactions. Creditors may not have perfect information about the financial linkages between their bank and the distressed bank, a scenario that is similar to the ones studied in Caballero and Simsek (2013) and Alvarez and Barlevy (2014). I investigate similar research questions to those considered by Alvarez and Barlevy (2014). They discuss whether information disclosure about distressed banks, who have incurred losses in a financial network, could be welfare improving. In contrast to Alvarez and Barlevy (2014), who emphasize the strategic decisions of outsiders in providing funding to finance the new investment opportunities, I show that information disclosure is rarely desirable. If the financial network is sufficiently diversified, the disclosure of the distressed bank's location has no effect on the extent of financial contagion or welfare. In an undiversified financial network, this information will trigger more panics from banks that are heavily connected to the initial distressed bank, and the dense financial connections will propagate this adverse impact into other banks. Contagious panics could make the entire system more fragile.

The theoretical literature on bank runs is based on the early work of Bryant (1980) and the classic model of Diamond and Dybvig (1983). Bank panics on single banks have been well studied in the literature. For example, Peck and Shell (2003) show that bank runs can occur when there is aggregate uncertainty about depositors' liquidity needs (even when a broad class of banking contracts is allowed as in Green and Lin (2003). Ennis and Keister (2010) and Ennis and Keister (2009) investigate this problem from the perspective of the government's limited commitment to suspend withdrawals. Uhlig (2010) shows how the assumptions of loss-averse investors and moral hazards could explain the bank panics that occurred during great recession.

In this paper, I model bank panics as coordination failures among each banks (patient) depositors, following Goldstein and Pauzner (2005). This approach is reasonable since the strategic complementarity in short-term creditors' withdrawal decisions is the key to the generation of panics. The model captures the sequential service constraint and no aggregate uncertainty is assumed.

Following Carlsson and Van Damme (1993) and Morris and Shin (2003), global game models have been widely adopted to study bank runs, currency attacks, and rollover risks. Examples can be found in Morris and Shin (1998, 2004), Rochet and Vives (2004), Goldstein and Pauzner (2004), Angeletos and Werning (2006), and Vives (2014). Adopting the global game setting, Dasgupta (2004) investigates how contagion can arise when banks have cross holdings of deposits and their liquidity shocks are correlated by considering a sequential game between two banks. His model features information spillovers, where the first bank run will have adverse effects on the beliefs of creditors about the second bank's liquidity position. By introducing a "model uncertainty" into a dynamic global game model, Chen and Suen (2016) rationalize the observation that market disturbances can spread across countries or regions that are not fundamentally linked. In their model, agents' beliefs about the state of the world can be dramatically revised after they observe a crisis occurring in another region. Hence, contagion can spread from one region to another unconnected one. Agents will attack more aggressively when expecting a world in a state of frenzy rather than a tranquil one.

In this paper, I analyze panic-driven bank runs in financial networks, where banks are linked via unsecured debt contracts. In my model, creditors take their actions simultaneously. Beyond their private noisy information about their bank's liquidity, creditors have no extra information about the state of the world or the strategic actions (or beliefs) of other creditors.

When considering the panic-based run on a single bank, Goldstein and Pauzner (2005) emphasize the lack of global strategic complementarity in the default regime in the standard Diamond and Dybivg model. They show that the global game setting can still be adopted to construct the unique equilibrium in that environment, and thus that the ex-ante probability of a bank run can be characterized. My model also features a lack of global strategic complementarity as this is a critical factor in determining how different network structures change creditors' incentives to run. Iachan and Nenov (2014) investigate how information quality will influence the probability of an individual bank run under a more general setting. Exploiting the lack of strategic complementarity in the setting of Goldstein and Pauzner (2005), Iachan and Nenov (2014) find that more precise private information increases the likelihood of panic-based bank runs. In this paper, I do not discuss the impact of information quality explicitly; however, the mechanism leading to my main result has some similarity with the one used in Iachan and Nenov (2014). I show that both diversifying connections and making liquidity shocks more correlated across banks will improve the private information about banks' counterparty risks, and thus trigger more panics in the financial market.

Outline

The remainder of the paper is organized as follows. In Section 1, I present the baseline model in a symmetric and regular network. In this section, I construct a unique equilibrium based on the symmetry of the network and show how this equilibrium depends on the network structure and the distribution of shocks. In Section 2, I extend the model to core periphery networks. I compare the financial soundness of the core banks and periphery banks in this section. In Section 3, I investigate another structure of exogenous shocks where, in addition to the liquidity shock to each bank, there is a sufficiently large shock to one bank to send this bank into distress ex-ante. I investigate the role that information about the financial linkages of the distressed bank plays in financial stability. In Section 4, I will discuss the robustness of the main results and their implications for network formation. Section 5 concludes. All omitted proofs and some additional technical lemmas are in the Appendix.

1 Model

Consider a single-product economy consisting of n banks, indexed by $i \in \mathcal{N} \equiv \{1, 2, \ldots, n\}$. Suppose that each bank has risk-neutral creditors of measure 2 and that each creditor is only associated with one bank. The economy lasts for three periods, $t = 0, 1, 2$.

At $t = 0$, each creditor is endowed with one unit of capital and deposits the full amount into one bank.[3] The deposit contract or short-term debt contract allows creditors to make withdrawals at $t = 1$ or $t = 2$. Creditors have uncertainties about their liquidity needs. The preference shock to creditors will be realized at $t = 1$.[4] Half of the creditors will be impatient, only valuing consumption at $t = 1$. The other half of the creditors will be patient, valuing consumption equally at $t = 1$ and $t = 2$.[5] Patient creditors $m \in [0, 1]$ of bank i need to decide whether to withdraw early at $t = 1$ ($a_{im} = 0$), or to withdraw late at $t = 2$ ($a_{im} = 0$). Let $w_i = \int_0^1 \mathbb{1}(a_{im} = 0)dm$ denote the share of bank i's patient creditors who decide to withdraw early.

In the initial period, each bank i makes a long-term investment with two units of capital raised from creditors. The riskless return from this long-term investment A_i will be realized at $t = 2$. Assume that the return from this long-term investment is sufficiently high to pay all the patient creditors who did not withdraw at $t = 1$. Bank i also has a legacy asset, which will generate a cash flow of $1 + \theta_i$ at $t = 1$.[6] I call θ_i the liquidity shock to bank i. Neither the long-term investment nor the legacy asset is pledgeable at $t = 1$. If a bank fails to meet its obligations at $t = 1$, both the long-term investment and the legacy asset will be liquidated at a recovery rate of λ. To simplify the analysis, I work with the extreme case where $\lambda = 0$[7]. Under this assumption, a bank becomes insolvent whenever the liquidation takes place.

Banks build financial connections through unsecured debt contracts in the initial period. Let k_{ij} denote the amount of capital borrowed by bank j from bank i. At $t = 1$, bank j needs to repay the face value $y_{ij} = R_{ij}k_{ij}$ to bank i, where R_{ij} is the corresponding interest rate. By definition, $y_{ij} > 0$ only if bank i makes a loan to bank j at $t = 0$ and $y_{ii} = 0$ for all $i \in \mathcal{N}$. Let $x_{ij} \in [0,\ y_{ij}]$

[3] Creditors in the model include the retail depositors, wholesale depositors, and other short-term creditors.

[4] Equivalently, creditors who make lending through unsecured short-term debts can be considered as depositors in the model. The short-term debt contract maturing at $t = 1$ will allow creditors to make rollover or withdrawal decisions in the intermediate period.

[5] There is no aggregate uncertainty in the preference shock and I assume the law of large numbers holds.

[6] Another way of interpreting $1 + \theta_i$ is as the outside funding that bank i can manage to obtain at $t = 1$ (or the total of the outside funding and liquid assets the bank holds).

[7] I will show later that this simplifying assumption that $\lambda = 0$ is not essential to any of my results.

denote the actual payment that bank j makes to bank i at $t = 1$. Note that if bank j does not have sufficient liquidity at $t = 1$ to fulfill its early withdrawals and service its interbank loans, $x_{ij} < y_{ij}$. The liquidity possessed by bank i to fulfill its liabilities at $t = 1$ is the cash flow from its legacy assets $1 + \theta_i$ and the actual repayment of interbank loans $\sum_{j \neq i} x_{ij}$ that it receives. Bank i needs to meet total withdrawals of $1 + w_i$ from creditors and repay the face value of its interbank loans of $y_i \equiv \sum_{j \neq i} y_{ji}$. To focus my analysis on the withdrawal decisions and interbank market clearing at $t = 1$, I take the lending and investment decisions of all banks at $t = 0$ as given.

Financial Networks

The interbank liabilities and counterparty relations can be represented by a financial network. Each *node* in the network represents a bank and a *directed edge* from bank i to bank j represents the interbank claim of bank i on bank j. A financial network of n banks is represented by a pair, $(\boldsymbol{Q}_{n \times n}, \boldsymbol{y})$[8], in which $\boldsymbol{Q}$ is the $n \times n$ relative liabilities matrix (or *weighted directed graph*) with:

$$\boldsymbol{Q}_{ij} = \begin{cases} \frac{y_{ij}}{y_j} & \text{if } y_j > 0 \\ 0 & \text{otherwise} \end{cases}$$

and $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)^T$ is the vector of interbank liabilities. The matrix $\boldsymbol{Q}_{n \times n}$ encodes the face value of the interbank liability of one bank to another as a proportion of the face value of the debtor bank's total interbank liabilities. By definition, $\sum_{i \neq j} Q_{ij} = 1$ for all $j \in \{1, 2, \ldots, n\}$. I first consider symmetric and regular financial networks. The definitions of regular and symmetric networks are as following.

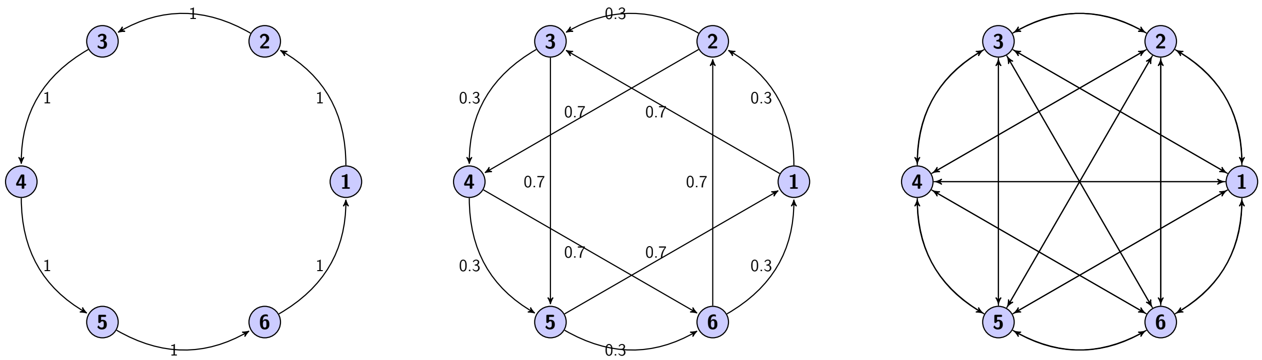
Definition 1.1. A financial network is *regular* if each bank has identical interbank liabilities and claims, i.e., $\sum_{i \neq j} y_{ij} = \sum_{j \neq i} y_{ji} = y$.

Definition 1.2. A financial network is *symmetric* (or, equivalently, circulant) if and only if

$$\forall i, j \in \mathcal{N}, \forall k \in \{1, 2, \ldots, n-1\}, \boldsymbol{Q}_{i,i+k \,(\text{mod } n)} = \boldsymbol{Q}_{j,j+k \,(\text{mod } n)}. \quad (1)$$ [9]

[8] In this paper, bold symbols are used to represent matrices or vectors.

[9] This definition of a symmetric network is equivalent to the definition of *circulant* network. Alvarez and Barlevy (2014) provide an example to show that a symmetric network (according to their definition) is not necessarily circulant. Here, for tractability, the focus is on circulant networks and they are considered symmetric.



A. Ring Network B. A Symmetric Network C. Complete Network

Figure 1: Examples of Symmetric Networks

In a regular financial network, each bank has the same amount of interbank liabilities and interbank claims, i.e., $y_i = y_j = y$ for all $i, j \in \mathcal{N}$. Hence, $\boldsymbol{y} = (y, y, \ldots, y)^T$. In a symmetric network, each bank is connected to the same number of counterparties and banks can be ranked in such a way that the distribution of interbank claims is identical for each bank. The graph $\boldsymbol{Q}$ is a circulant matrix, which means that $\boldsymbol{Q}_{ij}$ only depends on the distance $i - j \pmod{n}$ between banks.

Figure 1 provides three simple examples of symmetric networks. Panel A represents a *ring network*, or circular network, of $n = 6$ banks. Each bank borrows only from its counterclockwise neighbor and lends only to its clockwise neighbor, i.e., $\boldsymbol{Q}_{i,i-1} = \boldsymbol{Q}_{1,n} = 1$. In panel B, each bank has to repay 30% of its total interbank liabilities to its first counterclockwise neighbor and 70% to its second counter clockwise neighbor, i.e., $\boldsymbol{Q}_{i,i-1} = \boldsymbol{Q}_{1,n} = 0.3$ and $\boldsymbol{Q}_{i,i-2} = \boldsymbol{Q}_{1,n-1} = \boldsymbol{Q}_{2,n} = 0.7$. Panel C represents a *complete network* in which the interbank liabilities of each bank are distributed equally between all other banks, i.e., $\boldsymbol{Q}_{ij} = \frac{1}{n-1}$ for all $i \neq j$.

Limited Liability and Seniority

Both non-bank creditors who withdraw early and creditor banks have claims on bank i's liquid assets at $t = 1$. Assume that non-bank creditors are senior creditors and creditor banks are junior creditors. All junior (senior) creditors are of equal seniority. If the bank cannot meet its senior liabilities, i.e., $1 + \theta_i + \sum_{j \neq i} x_{ij} < 1 + w_i$, the proceeds will be distributed evenly among the non-bank creditors who decide to withdraw early. If the bank can meet the withdrawals, but fails to repay the interbank loans, i.e., $1 + w_i \leq 1 + \theta_i + \sum_{j \neq i} x_{ij} < 1 + w_i + y_i$, the creditor banks will be repaid in proportion to the face value of the interbank loans. Due to limited liability, bank j's repayment to bank i is

$$x_{ij} = \frac{y_{ij}}{y_j}\left[\min\{\theta_j - w_j + \sum_{i \neq k} x_{ki},\ y_j\}\right]^+,$$

where $[.]^+$ denotes $\max\{., 0\}$. From the above equation, the clearing payment made by bank j is the minimum of "what bank j has (after meeting the withdrawals)," i.e., $\theta_j - w_j + \sum_{i \neq k} x_{ki}$ and "what it owes," i.e., y_j. The amount that bank j pays to bank i is a face-value-weighted share ($\frac{y_{ij}}{y_j}$) of its total repayment of the interbank loans.

Let $x_i \equiv \sum_{k \neq i} x_{ki}$ be the actual repayment made by bank i to its creditor banks and let $\boldsymbol{x} \equiv (x_1, x_2, \ldots, x_n)^T$ represent *the clearing payment vector*. By definition, $x_{ij} = Q_{ij} x_j$. Let $e_i \equiv \theta_i - w_i$ denote bank $i's$ residual liquidity after meeting senior creditors' withdrawals and let $\boldsymbol{e} \equiv (e_1, e_2, \ldots, e_n)^T$ be the vector of residual liquidities.

Definition 2. Let $\{\theta_i\}_{i=1}^n$ be realizations of liquidity, $\{w_i\}_{i=1}^n$ be a realization of the patient creditors' aggregate withdrawal, and $(\boldsymbol{Q}, \boldsymbol{y})$ be a financial network. Then the *clearing payment vector* $\boldsymbol{x}$ is a fixed point of the mapping $\Phi(\boldsymbol{x}; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) : \prod_1^n [0, y_i] \to \prod_1^n [0, y_i]$, which satisfies

$$\Phi(\boldsymbol{x}; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) = [\min\{\boldsymbol{e} + \boldsymbol{Q}\boldsymbol{x}, \boldsymbol{y}\}]^+. \tag{2}$$

The notion of the clearing payment vector was introduced by Eisenberg and Noe (2001). The clearing payments in an interbank network are interdependent and mutually consistent. This definition is consistent with Acemoglu et al. (2015b), where $\boldsymbol{e}$ can be negative. Up to now, each bank's withdrawal $\{w_i\}_{i=1}^n$ has been taken as given. However, in this paper I focus on the strategic withdrawal decisions made by creditors. Creditors' withdrawal strategies will determine the clearing payments of interbank loans and the total liquidity available in the financial system, which is a crucial factor affecting the financial fragility of the interbank market.

Strategic Withdrawal Decisions

On entering the intermediate period, creditors must first decide whether they have an immediate need of liquidity.[10] Creditors have public knowledge of the financial network, but they do not

[10] Impatient creditors who have liquidity needs at $t = 1$ will withdraw early to fulfill their liquidity needs, no matter what information they receive. Hence, my analysis is focused on the strategic decisions of patient creditors. If the

have perfect knowledge about bank i's liquidity. For each bank $j \in \mathcal{N}$, nature picks θ_j from the well-known uniform prior $U[\underline{\theta}, \bar{\theta}]$. Patient creditor $m \in [0,1]$ of bank i will have some better information about their bank's liquidity shock θ_i.[11] They learn that from her private noisy information, $s_{im} = \theta_i + \sigma\epsilon_{im}$, where the error term ϵ_{im} is independent across creditors and uniformly distributed according to $U[-\frac{1}{2}, \frac{1}{2}]$.[12] The standard deviation of the noise is denoted by σ.

I assume that $\bar{\theta} > 1 + \sigma$ and $\underline{\theta} < -\sigma$. This assumption guarantees that the common prior is uninformative when agents have private information about θ_i. For a given $s_i \in [-\frac{\sigma}{2}, 1 + \frac{\sigma}{2}]$, the liquidity θ_i is uniformly distributed over $[s_i - \frac{1}{2}\sigma,\ s_i + \frac{1}{2}\sigma]$. Creditors understand that $\theta_i < 0$ (or $\theta_i > 1$) when their private information is $s_i < -\frac{\sigma}{2}$ (or $s_i > 1 + \frac{\sigma}{2}$, respectively).

A patient creditor m of bank i decides whether to withdraw early ($a_{im} = 0$) or delay ($a_{im} = 1$) at $t = 1$. If the bank remains solvent at $t = 2$, A_i is sufficiently large to guarantee that all remaining creditors will successfully receive $1 + r$. In the case where bank i defaults on its liabilities at $t = 1$, i.e., $e_i + \sum_{k \neq i} x_{ik} < y_i$, creditors who withdraw early will split the bank's liquid assets up to the principal value of 1 and all remaining creditors will receive nothing.[13] The payoff $u(a_{im}, w_i, \theta_i, \boldsymbol{Q}, \boldsymbol{y})$ is defined as follows:

$$
\begin{aligned}
u(a_{im} = 0, w_i, \theta_i, \boldsymbol{Q}, \boldsymbol{y}) &= \begin{cases} 1 & e_i + \sum_{j \neq i} x_{ij} \geq y_i \\ \min\left\{1, \frac{\theta_i + \sum_{k \neq i} x_{ik} + 1}{w_i + 1}\right\} & e_i + \sum_{j \neq i} x_{ij} < y_i \end{cases}, \\
u(a_{im} = 1, w_i, \theta_i, \boldsymbol{Q}, \boldsymbol{y}) &= \begin{cases} 1 + r & e_i + \sum_{k \neq i} x_{ik} \geq y_i \\ 0 & e_i + \sum_{k \neq i} x_{ik} < y_i. \end{cases}
\end{aligned} \tag{3}
$$

Discussion

The main aim of this paper is to investigate how the endogenous panics among creditors affect the fragility of the financial network. Several simplifying assumptions have been made in order to keep the model tractable. In the model, creditor banks are not allowed to delay the redemption of their

term, "creditors" is used without qualification, it shall refer to patient creditors.

[11] This assumption can be justified as creditors make deposits in bank located in their area, and they have some local information about this bank's financial strength.

[12] See Judd (1985) for the existence of a continuum of independent random variables.

[13] Once a bank defaults, all creditors who withdrew earlier as well as creditor banks become "residual claimants". The bank will honor their claims based on seniority and the face value of debts. However, Sequential Service Constraint can be incorporated if we consider each earlier withdrawer will receive 1 with probability $\min\left\{1, \frac{\theta_i + \sum_{k \neq i} x_{ik} + 1}{w_i + 1}\right\}$.

interbank claims and all interbank loans must be repaid at $t = 1$. This assumption allows me to focus my analysis on the withdrawal decisions of creditors.

The redemption of interbank claims makes the network effective for the analysis of systemic risk or financial contagion, as in other models of interbank networks (Eisenberg and Noe, 2001; Acemoglu et al., 2015b). This assumption also captures the fact that when a crisis is likely to hit, banks facing counterparty risk will be sufficiently cautious to request immediate repayment from their counterparties.[14] In my model, waiting to redeem interbank claims could be very costly for banks since they may have to liquidate their long-term profitable investments to meet all of their obligations. It is also more risky since banks' counterparties may default and nothing will be left for future redemption.

I also assume that the liquidation value of the long-term investments is $\lambda = 0$. Hence, banks cannot partially liquidate their long-term investments to meet their obligations at $t = 1$. Another simplifying assumption is that the cash flow at $t = 2$ is sufficiently high so that as long as there is no default at $t = 1$, all remaining creditors will earn interest. These assumptions highlight the coordination problem between patient creditors at $t = 1$. If they can successfully coordinate not panicking and withdrawing early, there will be no bank run and each of them will receive a higher payoff. Although these assumptions simplify the situation, they are not essential to my results. I will later show that my main results can be extended to include partial liquidation with $\lambda > 0$ and limited future cash flows.

Another simplifying assumption is the seniority of non-bank creditors over other general creditors. Recall that the term, "creditors" refers to retail depositors, wholesale depositors, and other short-term creditors in my model. In general, priority in bankruptcy was granted to depositors in 1993 under the term "deposit preference" in the United States (Marino and Bennett, 1999; Birchler, 2000).[15] However, the situation is more complicated for uninsured depositors. The federal laws give uninsured depositors the same priority as bond holders and other creditors, while some states adopt a deposit preference and give priority to uninsured depositors over the general creditors (Danisewicz et al., 2015). This assumption simplifies the model, but does not change the strategic complemen-

[14] The reason could be that institutional investors or creditors are more skillful and can make more informed decisions. For example, Rochet and Tirole (1996) argues that, when compared to non-bank creditors, banks are more capable of monitoring the risks and performance of their peers.

[15] On August 10, 1993, the United States enacted amendments to the Federal Deposit Insurance Act that created a preference for depositors in the distribution of the assets of a failed bank (the Federal Deposit Insurance Act (12 U.S.C. 1821(d)(11))).

tarity among creditors. I will later discuss the mechanism by which the main driving force of this model will still work when interbank loans are of equal seniority to bank deposits.

Equilibrium

In this section, I focus my analysis on symmetric and regular networks. Before discussing the relation between network structure and financial fragility, I first define the equilibrium and proceed to prove the existence and uniqueness of the equilibrium for any symmetric and regular network.

Definition 3. For a given financial network $(\boldsymbol{Q}, \boldsymbol{y})$, the *Bayesian Nash Equilibrium* is defined to be a collection of n withdrawal sets $\{\mathbb{S}_i\}_{i=1}^n$. Patient creditors of bank i will withdraw early if and only if they receive private information $s_i \in \mathbb{S}_i$. The Bayesian Nash Equilibrium $\boldsymbol{\mathcal{S}} \equiv \{\mathbb{S}_i\}_{i=1}^n$ satisfies the following conditions:

1. For given realizations of $\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \ldots, \theta_n)^T$ and all other creditors' strategies $\boldsymbol{\mathcal{S}}$ such that $e_i = \theta_i - w_i = \theta_i - Pr(s_i \in \mathbb{S}_i|\theta_i)$, the vector $\boldsymbol{x}(\boldsymbol{e}(\boldsymbol{\mathcal{S}}, \boldsymbol{\theta}), \boldsymbol{Q}, \boldsymbol{y})$ is the clearing payment vector satisfying Equation (2).

2. For a given $\boldsymbol{\mathcal{S}}$, for all $i \in \{1, 2, ..., n\}$, and all $s_i \in \mathbb{S}_i$, the expected payoff difference is

$$\begin{aligned} H_i(s_i, \boldsymbol{\mathcal{S}}) &= \int_{\boldsymbol{\theta} \in \{\boldsymbol{\theta}|x_i(\boldsymbol{e}(\boldsymbol{\mathcal{S}},\boldsymbol{\theta}),\boldsymbol{Q},\boldsymbol{y})=y\}} r dF(\boldsymbol{\theta}|s_i) + \int_{\boldsymbol{\theta} \in \{\boldsymbol{\theta}|0<x_i<y\}} (-1) dF(\boldsymbol{\theta}|s_i) \\ &\quad + \int_{\boldsymbol{\theta} \in \{\boldsymbol{\theta}|x_i=0\}} \left(-\frac{\theta_i + (\boldsymbol{Q}\boldsymbol{x})_i + 1}{w_i + 1}\right) dF(\boldsymbol{\theta}|s_i) < 0, \end{aligned} \tag{4}$$

   where $F(\boldsymbol{\theta}|s_i)$ denotes the cumulative distribution function (CDF) of $\boldsymbol{\theta}$, given s_i. Moreover, for all $s_i \in \mathbb{S}_i^C = \bar{\mathbb{S}} \setminus \mathbb{S}_i$,[16]

$$H_i(s_i, \boldsymbol{\mathcal{S}}) \geq 0.$$

For given creditors' withdrawal decisions, and any realization of banks' liquidity shocks, the vector $\boldsymbol{x}$ clears all payments between banks. The amount bank i is able to repay to its creditor banks depends on the total of the repayments that bank i can receive, i.e., $\sum_{k \neq i} x_{ik}$, the realization of liquidity shock θ_i, and the strategies taken by the creditors $\{\mathbb{S}_i\}_{i=1}^n$. Hence, $\boldsymbol{x}$ is a function of $\boldsymbol{\theta}$ and $\boldsymbol{\mathcal{S}}$. Since the mapping Φ defined in Equation (2) is a function of the network topology, the

[16] $\underline{s}(\bar{s})$ is the lower(upper) bound of private information. $\underline{s} = \underline{\theta} - \frac{\sigma}{2}$, $\bar{s} = \bar{\theta} + \frac{\sigma}{2}$ and $\bar{\mathbb{S}} = [\underline{s}, \bar{s}]$.

clearing payment vector $\boldsymbol{x}$ also depends on the network structure. The noisy private information s_i will help creditors to learn about bank i's liquidity shock θ_i. Creditors will rely on their prior beliefs about other banks' liquidity shocks and take all possible realizations into account.

Equation (4) represents the difference between the expected payoff of not withdrawing and that of withdrawing early. The necessary and sufficient condition for bank i to default is $x_i < y$, in which case this bank is unable to repay its interbank loans in full and thus defaults on its junior debt. If $x_i > 0$, the creditors who decide to withdraw early can still have their principal returned as senior creditors. Only if $x_i = 0$, will bank i default on its senior liabilities and each creditor who withdrew early will have only a part of their principal returned. For this network structure, distribution of liquidity shocks, and other creditors' strategies, a creditor with private information s_i will withdraw early only if the expected payoff difference $H(s_i, \mathcal{S})$ is negative.

Following Acemoglu et al. (2015b), I first show that the clearing payment vector $\boldsymbol{x}$ is generically unique when $\{\theta_i\}_{i=1}^n$ is continuously and uniformly distributed.

Lemma 1. When $\theta_i \sim U[\underline{\theta}, \bar{\theta}]$, $\boldsymbol{x}(\boldsymbol{e}, \boldsymbol{Q}, \boldsymbol{y})$ exists and is generically unique.

Now consider bank i's creditors. If the strategies of other banks' creditors $\{\mathbb{S}_{j\neq i}\}$ are given, then Lemma 2 shows that when the total interbank lending y is relatively low, the only rationalizable strategy for bank i's creditors is to withdraw early when their private information is lower than a threshold $s_i^\star(\{\mathbb{S}_{j\neq i}\})$. The proof uses an argument similar to the one given for a single (isolated) bank run model in Goldstein and Pauzner (2005). When a creditor receives private information $s_i > 1 + y + \frac{\sigma}{2}$, she understands that bank i will be able to meet all of its obligations at $t = 1$, even if all other patient creditors run on bank i and all counterparties fail to repay their interbank loans. This constitutes the upper dominance region. If creditor's private information is in upper dominance region, she will not withdraw early irrespective of what others will do. Hence, each creditor of bank i will believe that no one will withdraw early if they receive information within the upper dominance region. The lower dominance region, where creditors will definitely run on the bank irrespective of others' actions, can be constructed in a similar way. Under some restrictions on the noise contained in private information, the iterated elimination of dominated strategies ensures a unique rationalizable strategy for each creditor.

Lemma 2. Let $(\boldsymbol{Q}, \boldsymbol{y})$ be a regular financial network. If $\sigma \leq \sigma_0 \equiv \frac{1+r}{\ln 2} - 1$[17] and $y \leq y_0 \equiv$

[17]In the model, I assume that the share of patient creditors is the same as the impatient creditors, and there are in

$\min\{-\underline{\theta}, \bar{\theta} - 1\}$, then for any given strategy profile of other banks' creditors $\{\mathbb{S}_{j \neq i}\}$, there exists a unique threshold $s_i^\star(\{\mathbb{S}_{j \neq i}\}) \in [\underline{s}, \bar{s}]$ such that the only rationalizable strategy for bank i's creditor m is to choose $a_{im} = 1$ if and only if $s_i \geq s_i^\star(\{\mathbb{S}_{j \neq i}\})$.

Note that Lemma 2 holds for any regular network structure, including networks that are not symmetric. It follows from Lemma 2 that, if there exists an equilibrium, each bank's creditors will play a threshold strategy in equilibrium. Hence, $\mathcal{S} = \{s_1^\star, s_2^\star, \ldots, s_n^\star\}$.

In the Proposition 1, I posit that there exists a unique equilibrium for the symmetric and regular network and that this equilibrium is symmetric, i.e., $s_1^\star = s_2^\star = \ldots = s_n^\star$. In a regular and symmetric financial network, the creditors of each bank face an identical problem ex-ante before the liquidity shock is realized. It is not surprising that the equilibrium is symmetric. An asymmetric equilibrium cannot exist in a symmetric network. Otherwise, a bank (name it as Bank A without loss of generality) whose creditors take the most aggressive withdrawal strategies would transmit the liquidity risk (from the creditors' panic) to its creditor bank(s). For that reason, some creditor bank (name it Bank B without loss of generality) of bank A will face more counterparty risk than bank A because of the aggressive attack taking by bank A's creditors. Hence, bank B's creditors should attack even more aggressively than bank A's creditors, which constructs a contradiction.

Proposition 1. There exists a unique equilibrium in any regular and symmetric network. This equilibrium is symmetric, i.e., $s_i^\star = s^\star \in [\underline{s}, \bar{s}]$ for all i.

By Proposition 1, there exists a uniform threshold $s^\star$ for all creditors. The extent of the panic among creditors is determined by $s^\star$ since the ex-ante probability of withdrawal for each creditor is $\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \underline{\theta}} Pr(s < s^\star | \theta) d\theta$. The global game setting allows me to link the self-fulfilling behavior to fundamentals. The extent of the panic will depend on the realization of liquidity shock θ_i ex-post, but a higher threshold $s^\star$ will increase the extent of the panic in expectation given the prior of θ_i.

Define $\theta^\star \equiv w(s^\star, \theta^\star) = \frac{s_i^\star + \frac{\sigma}{2}}{1+\sigma}$. Whenever $\theta_i < \theta^\star$, bank i will default, even if bank i receives a full repayment of its interbank loans, as it will be unable to repay its outstanding interbank loans.[18]

total measure two creditors. If assuming there are impatient creditors of measure β_1 and patient creditors of measure β_2, the constraint will be $\sigma \leq \frac{1+r}{\ln(1+\frac{\beta_2}{\beta_1})}\beta_2 - 1$. This constraint on the precision of private information can be relaxed when there are more impatient creditors. From now on, if the condition on σ is not specified, σ will be fixed to be less than σ_0 .

[18]When $\theta_i \leq \theta^\star$, the share of creditors who withdraw early is weakly greater than the liquid asset holding of the bank, i.e., $1 + w(\theta, s^\star) \geq 1 + \theta_i$. Bank i's liquidity $1 + \theta_i + \sum_{j \neq i} x_{ij}$ is lower than the outstanding liabilities $1 + w_i + y$ since $\sum_{j \neq i} x_{ij} \leq y$.

If $\theta_i \geq \theta^\star$ for all $i \in \mathcal{N}$, it is easy to see that all banks will be able to repay their interbank loans, thus no bank will default. In my model, $\theta < \theta^\star$ is a sufficient, but not necessary, condition for a bank run to occur. When other banks receive bad liquidity shocks and fail to repay their interbank loans, bank i could default even if $\theta \geq \theta^\star$. This differs from the situation for the single bank run problem described in Goldstein and Pauzner (2005). In my model where bank i are interconnected, whether one bank will default or not depends on the realization of all liquidity shocks $\boldsymbol{\theta}$ but not only θ_i. However, I will show that the likelihood of default for each bank (before the liquidity shock realizes) in the symmetric network is linked to the equilibrium threshold $\theta^\star$. The definition of financial fragility is based on this probability. I will first define the notion of financial fragility and then compare different network structures with respect to this criterion.

Definition 4.1. A regular and symmetric financial network $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ is more *fragile* than $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ if for each bank i, the *ex-ante* probability of default, or $\mathbb{E}[\mathbb{1}(x_i < y)]$, is higher under $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ than under $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$.

Since both creditors and banks are assumed to be risk-neutral in the model, risk sharing between impatient creditors and patient creditors is not my main concern in the model. The only friction in the model that could induce welfare loss arises from the liquidation of long-term projects. The expected welfare loss is $\mathbb{E}[\sum_{i=1}^n \mathbb{1}(x_i < y)A_i]$. When the financial network is more fragile, the ex-ante probability of an costly liquidation will increase and thus induce more welfare loss.

Given the distribution of the liquidity shock θ_i to each bank, the equilibrium $s^\star$ can be used to find each bank's ex-ante probability of default. The definition of fragility given in Definition 4.1 is based on this ex-ante probability. An alternative, but equivalent, way of defining fragility is by comparing the sizes of the exogenous shocks that are needed to trigger a certain number of defaults (see, for example, the discussion in Cabrales et al. (2015)). However, this criterion is more difficult to apply here since there are n shocks in the model and a bank's solvency depends, not only on its own liquidity shock, but also on shocks to other banks.

I now define the stronger notion of *absolute fragility*.

Definition 4.2. The financial network $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ is absolutely more fragile than $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ if for any realization of $\{\theta_i\}_{i=1}^n$ that makes bank i default in financial network $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$, bank i will definitely default for $\{\theta_i\}_{i=1}^n$ in the financial network $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$.

In Proposition 2, I compare networks (with the same $\boldsymbol{Q}$, but different total interbank lending $\boldsymbol{y}$) using this definition of absolute fragility. I show that if one bank defaults under certain realization of $\boldsymbol{\theta}$ in symmetric and regular financial network $(\boldsymbol{Q}, \boldsymbol{y})$, this bank will definitely default under any symmetric and regular financial network $(\boldsymbol{Q}, \boldsymbol{y'})$ satisfying $\boldsymbol{y'} > \boldsymbol{y'}$. In other words, financial network with greater connections, i.e., each bank lends more to other banks (and borrows more from other banks), is more fragile.

Proposition 2. Let $(\boldsymbol{Q}, \boldsymbol{y}^1)$ and $(\boldsymbol{Q}, \boldsymbol{y}^2)$ be symmetric and regular financial networks. Then $(\boldsymbol{Q}, \boldsymbol{y}^1)$ is (absolutely) more fragile than $(\boldsymbol{Q}, \boldsymbol{y}^2)$ if $\boldsymbol{y}^2 < \boldsymbol{y}^1 \leq y_0 \times \mathbf{1}$.

For a given network $\boldsymbol{Q}$, if banks have a greater exposure to the interbank market, the financial network is more fragile. For each bank i in a regular network, the total repayment from its debtor banks is $\sum_{k\neq i} x_{ik}$, while the face value of its interbank loans is y. Increasing y will increase the face value of the liability of each bank, and will potentially also increase the repayments from other banks. The fact that the clearing payment vector $\boldsymbol{x}$ is a fixed point of $\Phi(\boldsymbol{x}) = [\min\{\boldsymbol{e} + \boldsymbol{Q}\boldsymbol{x}, \boldsymbol{y}\}]^+$ implies that, *ceteris paribus*, the increase in the repayments from other banks cannot be higher than the increase in y.[19] However, even if the increase in $\sum_{k\neq i} x_{ik}$ is identical to the increase in y, creditors will still have more incentives to withdraw because of the seniority of deposits.[20] Proposition 2 aligns with the results of Nier et al. (2007) and Acemoglu et al. (2015b),[21] who find that a relatively small increase in the connectivity will increase the contagion effect. However, there is no ex-ante insolvency problem in my model. Higher connectivity will make the financial system more fragile because it provides creditors with more incentives to run.

In Proposition 2, I compare the fragilities of financial networks based on the notion of absolute fragility in Definition 4.2. However, it is not possible to make comparisons of networks with different network structure $\boldsymbol{Q}$ based on this criterion. The reason for this is that when the topology of liquidity shocks is different, it is possible for a bank to default in the less fragile network under a certain realization of liquidity shocks, while remaining solvent in the more fragile network. However, I will

[19] The result is from the operator $\max\{., 0\}$. There are some states where bank i cannot services its senior liabilities, i.e., $e_i + \sum_{j\neq i} x_{ij} < 0$. In those states, even if all counterparties pay in full and bank i receives Δy more repayment when the interbank lending is increased by Δy, bank i will use this additional liquidity to pay its senior creditors and cannot pay Δy more to its junior creditors, or creditor banks. For details, see the proof in the Appendix.

[20] A similar mechanism works in the case where deposits and interbank loans are of equal seniority. Creditors will have a stronger incentive to run if they understand that other banks have more claims on the debtor bank's liquid assets when it goes bankrupt.

[21] In Nier et al. (2007), connectivity is defined as the probability that one bank made interbank loans to the other banks.

be able to judge the relative financial fragility of these networks, based on the *ex-ante* probability of default as in Definition 4.1.

Next, I fix the total interbank lending $\boldsymbol{y}$ and compare the financial fragility of various symmetric and regular networks. I first introduce a partial ordering of symmetric and regular networks based on the idea of diversifying connections. For bank i in a symmetric financial network, let C_i denote the set of banks on which bank i has interbank claims, i.e., $C_i \equiv \{j \neq i | \boldsymbol{Q}_{ij} > 0\}$, and denote the distribution of bank i's interbank claims by $D_i(\boldsymbol{Q})$, i.e., $D_i(\boldsymbol{Q}) \equiv \{\boldsymbol{Q}_{ij}\}_{j \in C_i}$. In a given symmetric financial network, each bank i has the same $|C_i|$ and $D_i(\boldsymbol{Q})$.[22] Hence, the *degree* of a symmetric network is $d(\boldsymbol{Q}) \equiv |C_i|$ (for all $i \in \mathcal{N}$). Let $D(\boldsymbol{Q})$ denote the weight distribution of the interbank loans for the symmetric financial network $\boldsymbol{Q}$. For example, the network in Panel (B) of Figure 1 has $d(\boldsymbol{Q}_B) = 2$ and $D(\boldsymbol{Q}_B) = \{0.7, 0.3\}$. Further, I define $\mathbb{Q}^n_m \equiv \{\boldsymbol{Q}_{n\times n} | d(\boldsymbol{Q}) = m\}$ to be the set of symmetric and regular networks (consisting of n banks) of degree m. In the following Definition 5, for any symmetric and regular network, I show that we can always find an equivalent one with a proper permutation.

Definition 5. Let $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ and $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ be symmetric and regular networks. Then $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ is *equivalent* to $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ if $\boldsymbol{y}^1 = \boldsymbol{y}^2$, $d(\boldsymbol{Q}^1) = d(\boldsymbol{Q}^2)$ and $D(\boldsymbol{Q}^1) = D(\boldsymbol{Q}^2)$.

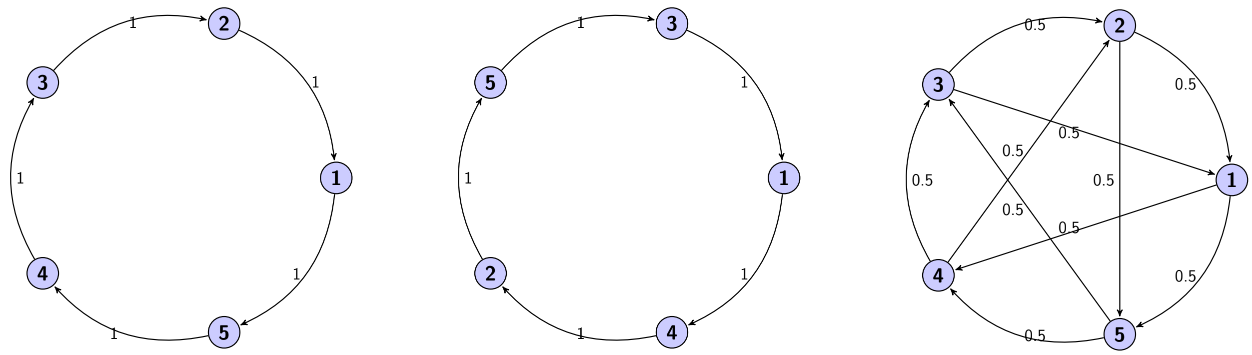
In a symmetric and regular network, reordering banks by a proper permutation (from $\mathcal{N}$ to $\mathcal{N}$) will change the weighted liability matrix $\boldsymbol{Q}$, but will have no real effect on the network topology since each bank is identical ex-ante. The equivalence of financial networks relies on this property. Given the network structure, assigning different numbers to the banks will have no impact on how the network transmits shocks. The partial ordering of more diversified networks is based on the convex combination of equivalent networks. The definition of convex combination is as following.

Definition 6. The financial network $(\boldsymbol{Q}^\gamma, \boldsymbol{y}^\gamma)$ is a γ-*convex combination* of financial networks $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ and $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ if there exists $\gamma \in [0, 1]$ such that $\boldsymbol{Q}^\gamma = \gamma \boldsymbol{Q}^1 + (1-\gamma)\boldsymbol{Q}^2$ and $\boldsymbol{y}^\gamma = \gamma \boldsymbol{y}^1 + (1-\gamma)\boldsymbol{y}^2$.

Lemma 3 shows that the properties of symmetry and regularity are preserved under convex combinations.

[22] Here $|S|$ denotes the cardinality of set S. By definition, the ordering of elements in $D_i(\boldsymbol{Q})$ does not matter.

Lemma 3. If networks $(\boldsymbol{Q}^1, \boldsymbol{y}^1)$ and $(\boldsymbol{Q}^2, \boldsymbol{y}^2)$ are both regular and symmetric, then any γ-convex combination of them is also regular and symmetric.



A. Ring Network 1 B. An Equivalent Ring Network C. Convex Combination of A and B

Figure 2: Example of Convex Combination

I consider convex combinations of equivalent symmetric and regular financial networks. By definition, the only difference between equivalent financial networks is the ordering of banks in the graph. Figure 2 presents an example of a convex combination of symmetric networks. The ring network in Panel A is equivalent to the ring network in Panel B. Since each bank is identical ex-ante, it is easy to see that each bank and its creditors will face the same risks in equivalent financial networks. Hence, equivalent networks will have the same equilibrium threshold $s^\star$. By Lemma 3, $\boldsymbol{Q}^C$ in Panel C of Figure 2, which is a γ-convex combination of the two equivalent ring networks $\boldsymbol{Q}^A$ and $\boldsymbol{Q}^B$ with $\gamma = 0.5$, is a regular and symmetric network. The formal definition of the ordering of more diversified networks is presented in Definition 7.

Definition 7. Let $(\boldsymbol{Q}^1, \boldsymbol{y})$ and $(\boldsymbol{Q}^2, \boldsymbol{y})$ be symmetric and regular financial networks. Then $(\boldsymbol{Q}^1, \boldsymbol{y})$ has *more diversified connections than* $(\boldsymbol{Q}^2, \boldsymbol{y})$, or $\boldsymbol{Q}^1 \succcurlyeq^d \boldsymbol{Q}^2$, if and only if $\boldsymbol{Q}^1$ is a γ-convex combination of $\boldsymbol{Q}^2$ and another symmetric and regular network that is equivalent to $\boldsymbol{Q}^2$.

Evidently, the financial network $\boldsymbol{Q}^C$ in Panel C of Figure 2 has more diversified connections than the ring networks in Panels A and B. The connections of a financial network can be diversified in two ways: (1) by increasing the number of connections of each bank (or increasing the degree $d(\boldsymbol{Q})$), (2) by distributing the same number of connections more evenly between the banks in the network (or $D(\boldsymbol{Q})$ is more evenly distributed). Proposition 3 shows that diversifying connections could make the financial system more fragile.

Proposition 3. Let $(\boldsymbol{Q}^1, \boldsymbol{y})$ and $(\boldsymbol{Q}^2, \boldsymbol{y})$ be symmetric and regular financial networks and suppose that $\boldsymbol{y} \leq y_0 \mathbf{1}$.

1. If $\boldsymbol{Q}^1 \succcurlyeq^d \boldsymbol{Q}^2$, then $s^\star(\boldsymbol{Q}^1, \boldsymbol{y}) \geq s^\star(\boldsymbol{Q}^2, \boldsymbol{y})$ and $\boldsymbol{Q}^1$ is more fragile than $\boldsymbol{Q}^2$.

2. The ring network, i.e., $\boldsymbol{Q} \in \mathbb{Q}_1$, is the least fragile symmetric network and the complete network, i.e., $\boldsymbol{Q}_{i,j} = \frac{1}{n-1}$ for all $j \neq i$, is the most fragile symmetric network.

3. Out of all financial networks in which each bank connects to m banks in a symmetric network of n banks, i.e., $\boldsymbol{Q} \in \mathbb{Q}_m$, those financial networks with evenly distributed connections, i.e., $D(\boldsymbol{Q}_0^m) = \{\frac{1}{m}\}$, are the most fragile.[23]

In Proposition 3, I posit that financial networks with more diversified connections are more prone to panic-based bank runs. When banks diversify their interbank loans to more debtor banks (or the interbank loans are more evenly distributed), they are essentially diversifying the risk that some debtor banks will be unable to repay their interbank loans in full. The distribution of the interbank repayments will be more centered around the mean than in those financial networks with less diversified connections.

As emphasized by Goldstein and Pauzner (2005), the strategic complementarity among creditors is missing in the regime of default. When a bank is in the default, creditors who withdraws early will split the bank's liquid assets evenly with other withdrawers. Each creditor's net incentive to run is lower when there are more creditors withdrawing early. Consequently, each creditor's net incentive to withdraw early is weakened when the bank defaults with lower liquid asset holdings. However, since the interest rate is fixed at r, as long as a bank does not go bankrupt, creditors have the same incentive to delay their withdrawals. The shift in the distribution of interbank repayments ($\sum_{k \neq i} x_{ik}$) lowers the probability of states in which banks have extremely low (high) liquidity ($1 + \theta_i + \sum_{k \neq i} x_{ik}$). More weight is assigned to the states in which the bank's liquidity is at an intermediate level. This change of distribution essentially shifts more of the probability density from states in which creditors have lower incentives to run to states in which the incentive to run is higher. Note that the incentive to run remains constant in the non-default regime. Hence, the shift in the distribution will create more panics among creditors and, consequently, make the system more fragile. The following example demonstrates this intuitive argument.

[23] Note that, for $m < n-1$, there could be multiple equivalent networks that are symmetric and regular, in which each bank is connected to m banks and each connection represents $\frac{1}{m}$ of the total interbank loans.

A Simple Example

It is fairly difficult to quantitatively investigate the fragility of financial networks in my model. Each bank is exposed to a liquidity shock and the interbank payment vector $\boldsymbol{x}$ is a fixed point for any possible realization of n liquidity shocks. In making their withdrawal decisions, creditors take all possible states into consideration. Here I provide a simple example to illustrate why more diversified network is more fragile. I simplify the model by assuming $\sigma \to 0$ and $y < \min\{\frac{1}{1+r}, y_0\}$[24] and generate solutions of the model for the ring network and the complete network.

The assumption that $\sigma \to 0$ is widely adopted in the global game literature, but it is restrictive in my model. When $\sigma \to 0$, creditors will coordinate their withdrawal decisions perfectly. If the liquidity realization θ_i is less than the equilibrium threshold $s^\star$, all creditors will run on the bank and $w_i = 1$. Otherwise, $w_i = 0$. Under the assumption that the total interbank lending is relatively low, i.e., $y < \min\{\frac{1}{1+r}, y_0\}$, the solvency of each bank in the financial network will not depend on how much its counterparties can repay. Although the equilibrium threshold still depends on the counterparty risk as this affects the payoffs received by creditors when taking different actions, this setting assume away the impact from the actions of the counterparties of a bank's counterparties. Under all simplied assumptions, each bank in the financial network will either repay the interbank loan in full or fail to pay anything to its creditor banks, independent of the network structure and how much interbank repayment it will receive for its debtor banks.

Figure 3 shows the expected payoff difference between withdrawing late and withdrawing early in equilibrium for the complete network. The payoff difference in the ring network is similary. It is easy to see that the incentives to withdraw early are weakened when banks have low liquidity in the regime of default. The incentives for creditors to withdraw late in the regime of non-default are constant because of the fixed interest rate.

In Figure 4, I present the distribution of the interbank repayments in equilibrium for both the ring network and the complete network. In the ring network, there are only two possibilities: either the sole debtor bank is insolvent and cannot repay anything, or it is solvent and will repay its debts in full. As discussed earlier, in the complete network, which has more diversified connections than the ring network, the distribution of interbank repayments is more centered around the mean.

[24] $\theta^\star = s^\star = \frac{1}{1+r}$ is the equilibrium when all interbank loans will be repaid in full and $\sigma \to 0$. So, in the model where the counterparties cannot fully repay the interbank loan in all states, the equilibrium threshold $s^\star$ is always higher than $\frac{1}{1+r}$.

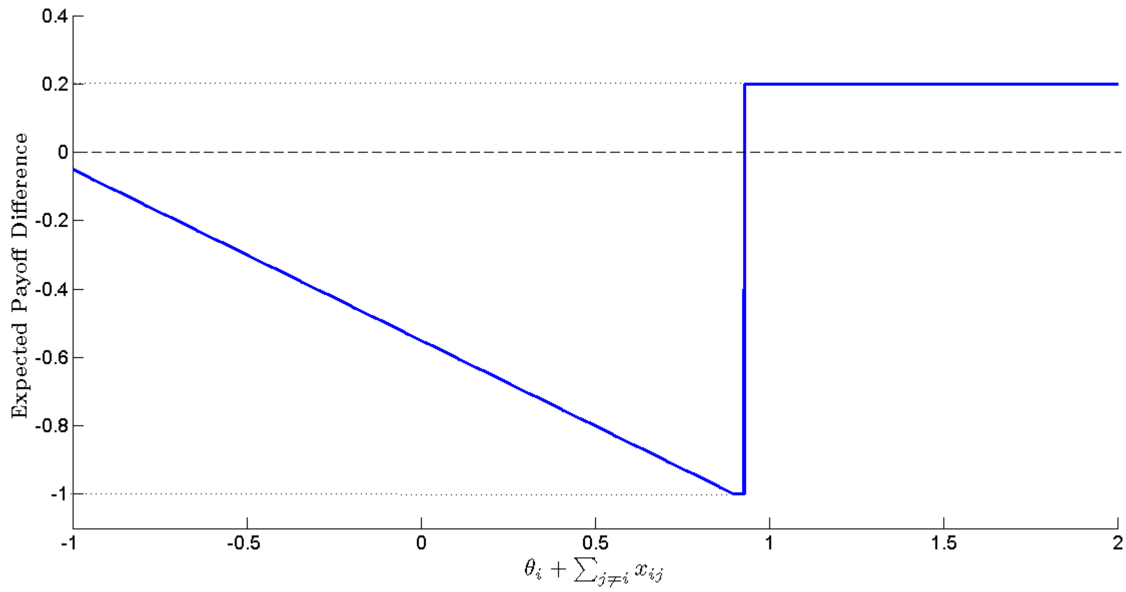


Figure 3: Expected Payoff Difference in Complete network ($n = 13, \bar{\theta} = 1.8, \underline{\theta} = -1, y = r = 0.2$)

Figure 5 presents the equilibrium threshold $s^\star$ as a function of the total interbank lending y. Regardless of the network structure, the equilibrium threshold $s^\star$ is an increasing function of y. This means that higher levels of interbank lending will make the financial system more fragile as in Proposition 2. The complete network has a higher equilibrium threshold $s^\star$ than the ring network and will generate more panics among creditors (Proposition 3). Figure 5 shows that the difference in $s^\star$ also increases with the total interbank lending y. The difference in the equilibrium thresholds $s^\star$ for the two networks is relatively small because my assumptions on σ and y remove the indirect effects from the counterparties of counterparties. Without these assumptions, the impact of diversified connections on financial fragility would be more significant.

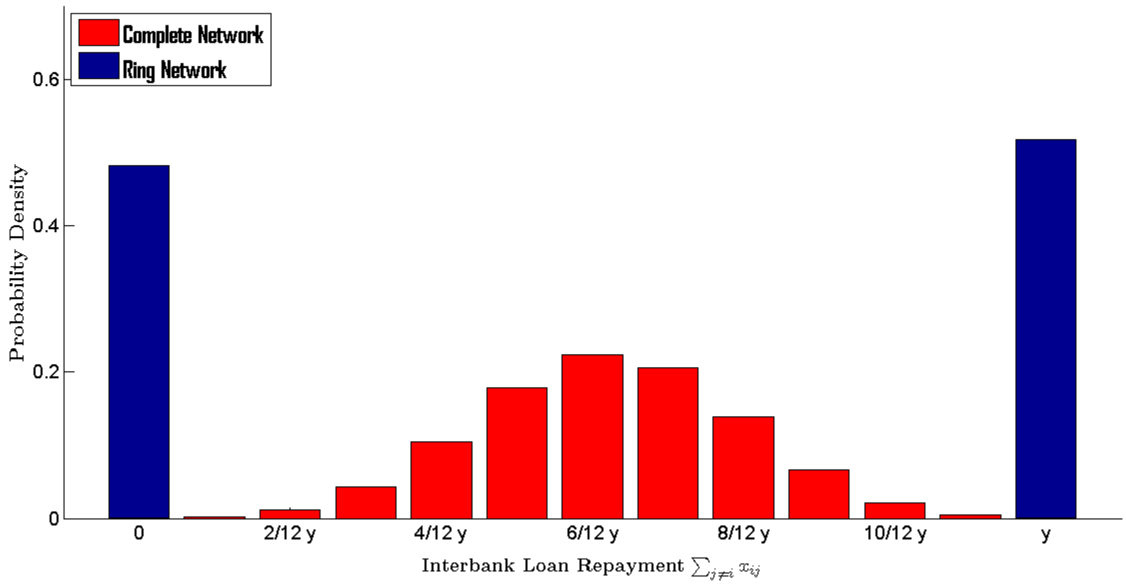


Figure 4: Distribution of Interbank Repayment: the Ring and Complete Networks

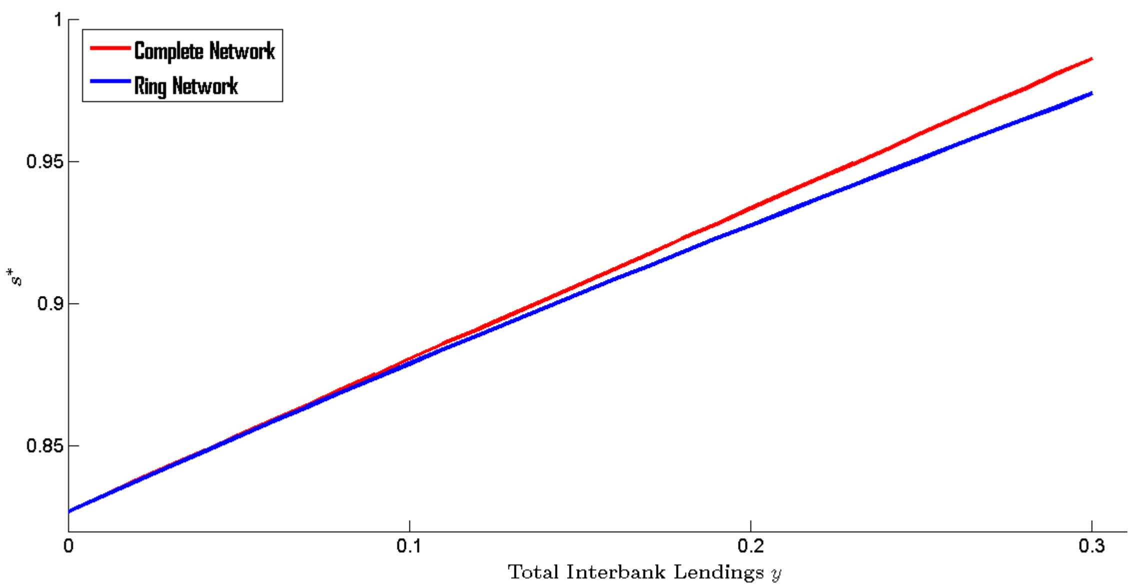


Figure 5: Equilibrium Threshold: the Ring and Complete Network

Correlated Liquidity Shocks

Up to now, I have assumed that liquidity shocks are idiosyncratic, i.e., θ_i and θ_j are mutually independent. Next I consider the case where each bank's liquidity shock contains an aggregate component and a idiosyncratic component. This allows me to investigate the impact of the correlation in liquidity shocks on the systematic risk. Suppose the liquidity shock is given by $\theta_i = \mu z + (1-\mu)\eta_i$, where z represents the aggregate shock and η_i represents the idiosyncratic shock. Assume that $z \sim N(0, \sigma_z^2)$, $\eta_i \sim N(0, \sigma_\eta^2)$.[25] I assume that η_i is independent and identically distributed across banks and the aggregate shock z is independent of the idiosyncratic shock η_i. Under these assumptions, the liquidities of different banks are correlated because of the aggregate component. The correlation coefficient

$$\kappa(\theta_i, \theta_j) = \frac{1}{1 + (\frac{1-\mu}{\mu})^2 \frac{\sigma_\eta^2}{\sigma_z^2}}$$

is increasing in the weight of the aggregate component μ. In Proposition 4, I posit that the financial network is more fragile if the liquidity shocks to different banks are more correlated (or the aggregate component has a higher weight in the idiosyncratic shock).

Proposition 4. Suppose $0 < \mu_2 < \mu_1 < 1$, $\kappa(\mu_1) > \kappa(\mu_2)$, and $s^\star(\mu_1) > s^\star(\mu_2)$. The financial network is more fragile if the weight of the aggregate shock forms a greater part of the individual bank's liquidity shock.

A higher weight μ for the aggregate shock will make the aggregate component z relatively more influential than the idiosyncratic shock η_i on the liquidity shock θ_i. In this case, the private information regarding θ_i will be more informative about the other banks' liquidities. Proposition 4 implies that when creditors have less information about other banks' liquidities, there will be less panic and the financial system will be more robust.

To understand the mechanism for this increase in fragility, consider the behavior of the marginal creditor who has received the equilibrium threshold information $s^\star$. She is indifferent between withdrawing early and waiting. If her information $s^\star$ is more informative about other banks' liquidities, she tends to believe that it is less likely for other banks to have extremely low or extremely high liquidity. This makes her belief about the distribution of the interbank repayment $\sum_{j \neq i} x_{ij}$ more

[25] In this subsection, I work with the improper prior, i.e., the uniform prior where $\underline{\theta} \to -\infty$ and $\bar{\theta} \to \infty$. Under this assumption, the uniform prior can be approximated by $N(0, \sigma_\theta^2)$, where $\sigma_\theta^2 = \sigma_z^2 + \sigma_\eta^2$. I first assume that σ_z and σ_η are both finite and that the ratio $\frac{\sigma_\eta}{\sigma_z}$ is fixed. I will take limits to infinity to approximate the improper prior

centered. For the reasons given in the discussion of Proposition 3, this more centered distribution of interbank repayments will incentivize marginal creditors to withdraw early. This is why creditors need to be more optimistic about the bank's liquidity to be indifferent, or $s^\star(\mu_1) > s^\star(\mu_2)$ if $\mu_1 > \mu_2$. A similar mechanism for the relation between information quality and the probability of bank runs for individual financial institutions can be found in Iachan and Nenov (2014).

As argued by Wagner (2010) and Ibragimov et al. (2011), financial institutions have incentives to diversify their portfolio holdings to hedge idiosyncratic risks, but they may overlook the externalities of holding similar portfolios to the systemic risk. However, based on Proposition 4, this paper finds that diversifying idiosyncratic shocks will not only increase the systemic risk in the financial system, but it also increases the liquidity risk for any individual bank because of the panic from its creditors.

It is worthwhile to note the difference between financial institutions diversifying their connections and diversifying their investment portfolios. Although I have shown that both of these could make the financial system more fragile during normal times, the underlying mechanisms are quite different.

In Proposition 3, I posit that financial institutions in a network with more diversified connections are essentially diversifying their counterparty risks, while each bank is still exposed to an idiosyncratic risk. Taking the network structure as given, Proposition 4 shows that if financial institutions diversify their investment portfolios and make the liquidity shocks more (positively) correlated, creditors with better information about other banks' liquidity risks will panic more, thereby increasing the fragility of the financial network.

To summarize, during normal times when each bank in the financial network is facing a dixed distribution of liquidity shock but there is no distressed bank before the shock realizes and outside creditors take their strategic decisions, a symmetric and regular financial network will be more fragile when each bank has higher greater exposure to the interbank market, or when the pattern of financial linkages between banks is more diversified. Based on these findings, the provision on "single counterparty exposure limits" in the Dodd-Frank Act (Section 165(e)), which attempts to prevent one institution's problems from spreading to the rest of the system by limiting each financial institution's exposure to any single counterparty, could be effective in promoting financial stability by restricting the aggregate exposure of each bank. However, it also provides incentives for financial institutions to build less dense and more diversified linkages, which could endogenously create more panics and undermine financial stability.

2 Core Periphery Networks

Empirical studies show that the core periphery network could better fit the data of the interbank market (e.g., Bech and Atalay (2010), van Lelyveld et al. (2014), Craig and Von Peter (2014)). In this section, I extend the baseline model to core periphery networks that are asymmetric and irregular. A financial network is a *core periphery network* if all banks in the network can be partitioned into two groups, i.e., the core banks and the periphery banks. Each bank in the core is linked to all other core banks and is connected to some periphery banks, and each bank in the periphery has no connection to any of the other periphery banks, but has at least one link to the core. A formal definition of core periphery network is as follows.

Definition 8. A financial network is a *core periphery network* if there exists a set $C \subset \mathcal{N}$ and a set $P = \mathcal{N} \backslash C$ such that all of the following conditions are satisfied:

1. $\boldsymbol{Q}_{ij} > 0$ for all $\{i, j\} \subset C$,
2. $\boldsymbol{Q}_{ij} = 0$ for all $\{i, j\} \subset P$,
3. For all $i \in P$, there is a unique $j \in C$ such that $\boldsymbol{Q}_{ij} = 1$. For all $k \in C$ with $k \neq j$, the value $\boldsymbol{Q}_{ik} = 0$.

I examine the fragility of the core periphery networks and the financial soundness of different banks in these asymmetric networks. In order to achieve this aim, I focus one those core periphery networks that have some degree of symmetry by assuming that all core (or periphery) banks are identical *ex-ante*. Suppose there are m core banks, i.e., $|\mathcal{C}| = m$. I will number the core banks from 1 to m, and the periphery banks from $m + 1$ to n in the network. The relative liability matrix $\boldsymbol{Q}$ then has the form

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_{CC} & \boldsymbol{Q}_{CP} \\ \boldsymbol{Q}_{PC} & \boldsymbol{Q}_{pp} \end{bmatrix}.$$

I assume that $\boldsymbol{Q}_{CC}$, which encodes the structure of the financial interconnections between core banks, is *symmetric* according to Definition 1.2. I also assume that each periphery bank borrows in the interbank market from one core bank and makes loans to one of the core banks,[26] i.e., for all $i \in \mathcal{P}$, there is a unique $j \in C$ such that $\boldsymbol{Q}_{ij} = 1$ and there is a unique $k \in \mathcal{C}$ such that

[26] In theory, the periphery bank can make loans to and borrow from the same core bank (at different times), or make loans to one bank and borrow from another.

$\boldsymbol{Q}_{ki} = 1$. In addition, for all $l \in \mathcal{C} \setminus \{j,k\}$, $\boldsymbol{Q}_{il} = \boldsymbol{Q}_{li} = 0$. Moreover, each core bank borrows from q periphery banks and makes loans to q periphery banks, i.e., for each core bank $j \in \mathcal{C}$, $|\{h \in \mathcal{P} : \boldsymbol{Q}_{jh} > 0\}| = |\{h \in \mathcal{P} : \boldsymbol{Q}_{hj} > 0\}| = q$. Further assume that the core periphery network is *regular* (see Definition 1.1), i.e., the total interbank borrowing and lending is identical across core banks (or periphery banks). The amount of total borrowing and lending for each periphery bank is y_p, and for each core bank is $qy_p + y_c$, where y_c is each core bank's total interbank borrowing from other core banks. Employing this notation, the core periphery network can be represented by the triple, $(\boldsymbol{Q}, y_c, y_p)$.

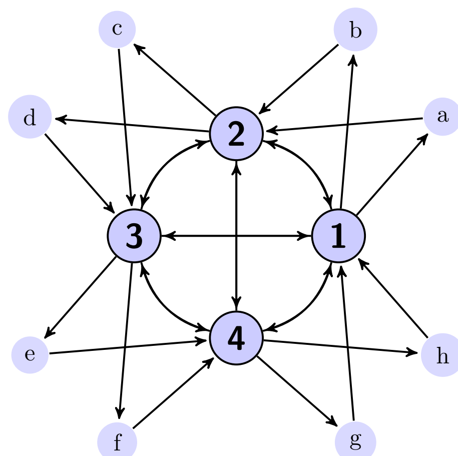


Figure 6: Example of a Core Periphery Network

As in the model of a symmetric and regular network, each bank is assumed to have the same distribution of liquidity shock, $\theta_i \in U[\underline{\theta}, \bar{\theta}]$. All other structures apart from the network structure, including each bank's balance sheet and the creditors' payoff structures, are assumed to be the same as in the model of a symmetric and regular network. These assumptions help us to maintain the symmetry of the core periphery network. Figure 6 is a simple example of a core periphery network that satisfies all of the above assumptions. In Figure 6, each core bank is linked to three periphery banks while each periphery bank is connected to one core bank.

I construct an equilibrium for a core periphery network as formulated in Definition 8. Note that this equilibrium might be asymmetric in the sense that the creditors of different core banks may have different threshold strategies because their strategies depend on the strategies of the creditors of some periphery banks, which, in turn, have no direct effect on the other core banks. For example, in the network presented in Figure 6, it is possible that the creditors of banks $1a$, $1b$, and $1c$ are more aggressive in withdrawing early than the creditors of banks $2a$, $2b$, and $2c$. In this case, bank 1's creditors will be more aggressive in withdrawing than bank 2' s creditors, which is consistent with the assumptions of the creditors' withdrawal strategies for the periphery banks. Hence, this system could have multiple equilibria. I will restrict my attention to the symmetric equilibrium, where all creditors of the core (periphery) banks play the threshold equilibrium $s_c^\star$ ($s_p^\star$). In Proposition 5, I posit that a core periphery network has a unique symmetric equilibrium.

Proposition 5. When $y_p < y_c < y_0$, there exists a unique symmetric equilibrium in which the creditors of core bank i will withdraw early if and only if $s_i < s_c^\star$ and the creditors of the periphery bank j will withdraw early if and only if $s_j < s_p^\star$.

In the core periphery network, core banks have higher exposures to interbank borrowing than periphery banks. Each of them borrows y_p from each periphery bank and borrows y_c from all other core banks. The larger number of connections and the higher amount of borrowing mean that core banks are exposed to higher counterparty risk than periphery banks. Proposition 6 shows that core banks will be more fragile than periphery banks when $y_c > y_p$, which means the total interbank borrowing from core banks will be greater than the borrowing from a single periphery bank.

Proposition 6. When $y_p \leq y_c$, in the symmetric equilibrium for any core periphery network, $s_c^\star > s_p^\star$, which means that the *ex-ante* probability of default is higher for the core banks.

In core periphery networks, according to Proposition 6, the more interconnected core does not only make core banks more fragile[27] to panic-based runs, but also contributes to the fragility of the whole system. This is because periphery banks face higher counterparty risk from the more fragile core banks. Up to now, it has been assumed that all banks in the financial network have the same distribution of liquid assets (or θ) and each bank has measure two of creditors. It is more realistic to allow banks to be heterogeneous in size and to assume that core banks have larger balance sheets than periphery banks.

Given the core periphery network $(\boldsymbol{Q}, y_c, y_p)$, I assume that the size of each core (periphery) bank is controlled by a parameter k_c (k_p), i.e., each core bank has liquid assets with value $k_c(1+\theta_i)$ and has creditors of measure $2k_c$. The corresponding values for periphery banks are $k_p(1+\theta_j)$ and

[27] Bank i is more *fragile* than bank j if bank i's ex-ante probability of default is higher than bank j's, see also Definition 4.1.

$2k_p$, respectively. Further, I assume that the total borrowing between core banks is $\tau_c y_c$ while the total borrowing of periphery banks from core banks is $\tau_p y_p$. The aim of this exercise is to understand how systemic risk depends on the size of the banks (funded by more deposits), as well as the size of the interbank loans. In Proposition 7, I posit the outcome of the results of a comparative statistical analysis. A core periphery network is more fragile if each bank in the system has a higher probability of default.

Proposition 7. Let $(\boldsymbol{Q}, \tau_c y_c, \tau_p y_p)$ be a core periphery network with core (periphery) banks of size k_c (k_p).[28] Then in the symmetric equilibrium,

1. Core banks are more fragile than periphery banks as long as $\frac{\tau_p}{k_p} y_p \leq \frac{\tau_c}{k_c} y_c$;
2. *Ceteris paribus*, the financial system is less fragile when k_c (or k_p) increases;
3. *Ceteris paribus*, the financial system is more fragile when τ_c (or τ_p) increases;
4. When $\frac{\tau_c}{k_c}$ and $\frac{\tau_p}{k_c}$ are fixed, increases in k_c will make the financial system more fragile.

Proposition 7 shows that as long as the connectivity of the core ($\frac{k_c}{\tau_c} y_c$) is higher than the periphery, the core banks will face more risk from the coordination game among creditors. When financial institutions raise funding from depositors or short-term creditors, but not from borrowing in the interbank market, the expansion in the balance sheet (corresponding to increasing k, but keeping τ fixed) can deter creditors from running on the bank. However, if banks borrow more from the interbank market (corresponding to an increase in τ), the risk of default caused by creditors' panic will be higher. The intuition behind these outcomes is that when banks rely more on the interbank market than on deposits, they will have a higher exposure to counterparty risk, which will cause more panics among creditors, thereby making the system more fragile. When core banks grow larger and intermediate more loans for periphery banks in the interbank market (corresponding to an increase in k_c and τ_p), periphery banks face higher counterparty risk ($\frac{\tau_p}{k_p}$ is higher). Hence, creditors will withdraw more aggressively from the periphery banks and this adverse effect will spread into the core banks as well, even though the creditors associated with the core banks initially face the same risk as before ($\frac{\tau_c}{k_c}$ and $\frac{\tau_c}{k_c}$ are both fixed).

[28]Note that the system is "constant return to scale" as can be seen from the creditor's payoff in Equation (3). Increasing all parameters, or k_c, k_p, τ_c, τ_p by a constant amount will not change the fragility of the financial network.

3 Information Disclosure and Financial Contagion

In the baseline model, each bank in the financial network faces a liquidity shock, but no bank will fail for sure in facing for a given distribution of liquidity shocks. The endogenously generated panics among creditors will determine whether bank will fall into crisis ex-post. In this section, I consider the case where there is one bank receiving a sufficiently large shock to make it insolvent ex-ante, independent of creditors' reactions. In this context, the financial network is likely to spread the collapse of the initial distressed bank to other directly and indirectly linked banks, and the panic will magnify the initial shock and facilitate financial contagion. If creditors do not have information about the location of the initial distressed bank, or they are uncertain about the financial linkage between their bank and the initial distressed bank, could information disclosure help to restore financial stability in financial networks? In this section, I examine how financial fragility depend on the policy of information disclosure, as well as the network structure.

Let $(\boldsymbol{Q}, \boldsymbol{y})$ be a financial network. I assume that, without loss of generality, there is a sufficiently large shock to make bank 1 default ex-ante, independent of the actions taken by market participants and other banks. The size of the liquidity shock satisfies $\theta_1 = \theta_l < \min\{\underline{\theta}, -y\}$.[29] In this case, even in the best scenario where all debtor banks repay their interbank loans in full, i.e., $\sum x_{1j} = y$ and no early withdrawals take place, i.e., $w_1 = 0$, bank 1 will still go bankrupt and be unable to repay any interbank loans. The value of the liquid assets that bank 1 has available to repay interbank loans is always negative, i.e., $\theta_1 + \sum_{j \neq 1} x_{1j} - w_1 < 0$.

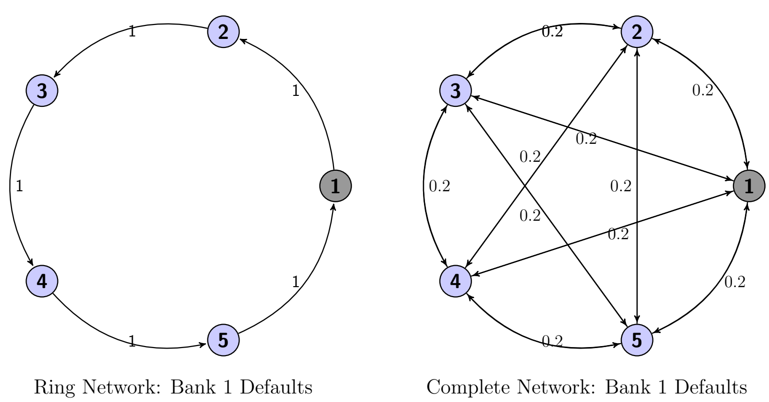


Figure 7: Distressed Case: Ring and Complete Network

[29] An alternative way to justify the liquidity shock could be that all creditors of bank 1 will run on bank 1 due to some "sunspot."

Incomplete Information

Before considering the impact of information disclosure, I first consider the benchmark case in which creditors do not have information about the financial connections between their bank and the initial distressed one. The creditors of bank 1 understand that their bank must default and they will all run on the bank. By contrast, the creditors of any bank $i \in \mathcal{N}\backslash\{1\}$ understand that a crisis is underway, but only have local information that their bank is not in distress. When there is a distressed bank in the system ex-ante, as in Caballero and Simsek (2013), the financial network might be too complicated for creditors to understand how their bank is linked to this distressed one, even if they have perfect information about the identity of the distressed bank.

In a financial network, information about the initial troubled bank is important because, without this information, creditors will be unable to predict the counterparty risk that their bank is facing. When this information is not available, I assume that each creditor will hold the *neutral* belief that each bank $j \in \mathcal{N}\backslash\{i\}$ has probability $\frac{1}{n-1}$ of being the defaulting bank. Otherwise, bank $j's$ liquidity θ_j will follow the uniform distribution $U[\underline{\theta}, \bar{\theta}]$ as in the baseline model. In this context, since the creditors of non-distressed bank assign equal probabilities to the likelihood of each bank being the distressed bank in the symmetric network (except the bank that is the direct counterparty of creditors), they are facing exactly the same problem. By Proposition 1, the equilibrium is still symmetric. Intuitively, the only difference from the baseline model is that θ_j is not distributed uniformly over $\underline{\theta}$ to $\bar{\theta}$.[30] The distributions of θ_i and $\theta_j (j \neq i)$ for bank $i's$ creditors are as follows:

$$\theta_i \sim U[\underline{\theta}, \bar{\theta}], \quad \theta_j = \begin{cases} \theta_l & \text{w.p.} \frac{1}{n-1} \\ \sim U[\underline{\theta}, \bar{\theta}] & \text{w.p.} \frac{n-2}{n-1}. \end{cases}$$

Hence, all results for the baseline model will carry over to this model. In Proposition 8, I summarize the results for the case that creditors hold neutral beliefs over the location of the distressed bank when such information is not available.

Proposition 8. Suppose creditors have no information about the financial linkage of their bank with the distressed bank, but hold neutral beliefs about its position in a symmetric and regular network. Then

[30] Note that for bank i's creditors, the shock θ_l only changes their prior beliefs about other banks' liquidity shocks, but not the distribution of θ_i. Beyond that, when $\theta_j = \theta_l$, bank j will pay nothing to its creditor banks, i.e., $x_j = 0$.

1. There is unique equilibrium characterized by $s_i^\star = s^\star$ for all banks $i \in \mathcal{N} \backslash \{1\}$. Patient creditors will withdraw early if and only if $s_i < s^\star$.

2. The symmetric and regular network $(\boldsymbol{Q}, \boldsymbol{y}_1)$ is more fragile than $(\boldsymbol{Q}, \boldsymbol{y}_2)$ if $\boldsymbol{y}_2 < \boldsymbol{y}_1 \leq y_0\mathbf{1}$.

3. The symmetric and regular network $(\boldsymbol{Q}_1, \boldsymbol{y})$ is more fragile than $(\boldsymbol{Q}_2, \boldsymbol{y})$ if $\boldsymbol{Q}_1$ has more diversified connections than $\boldsymbol{Q}_2$.

When such a crisis is underway and one bank is already in trouble, creditors will have lower expectations of the liquidity of other banks and will be more likely to withdraw their funds. Note that, in contrast to other models of financial contagion, the likelihood of a cascade of defaults depends on the realizations of the liquidity shocks to other banks and the panics of creditors in equilibrium. When creditors hold *neutral* beliefs, they take all possible consequences into consideration and do not panic as much after the first collapse. The lack of information, to some extent, helps to avoid cascade of defaults.

However, in the extreme case where creditors think about the worst possible case, as in Caballero and Simsek (2013) or they hold *cautious* beliefs (i.e., they believe that the troubled bank will be the (largest) direct counterparty of their bank), the less diversified network could be extremely fragile and financial contagion would occur.[31] When the interbank network is undiversified and the financial linkages are dense, creditors will make an aggressive run in their banks when they hold the cautious belief that the troubled bank is the (largest) direct counterparty of their bank. By contrast, in financial networks with more diversified connections, this belief will induce less panic because the initial troubled bank represents only a small fraction of the counterparty risk. This argument is formalized in Proposition 9.

Proposition 9. If creditors have no information about the financial linkage of their bank with the initially distressed bank, but hold cautious beliefs about its position in the network, the ring network is the most fragile symmetric and regular network (when $\boldsymbol{y} < y_0\mathbf{1}$).

It is easy to see that, when creditors are uncertain about the position of the troubled bank, their beliefs will play an important role in determining the extent of financial contagion. I show

[31] Caballero and Simsek (2013) assume that all market participants will be cautious and believe that the defaulting bank is the neighbor of their counterparties. This assumption, although plausible in practice, is *irrational* in theory. It could be self-fulfilling that contagion with all banks defaulting is more likely to happen under this belief, but there still exist states where certain banks will remain solvent when the realization of θ is sufficiently high.

that information disclosure can have very different impacts on the spread of financial contagion for different network structures, depending on whether creditors hold neutral or cautious beliefs about the position of the troubled bank.

Information Disclosure: Complete Information

I next consider the case where all creditors have perfect information about the financial linkages with the distressed bank. This could happen when banks disclose this information voluntarily or mandatorily as required by regulators. The availability of such information will eliminate the uncertainty about the location of the distressed bank.

I mainly consider two network structures in this context: the ring network and the complete network. If the financial network is perfectly diversified as in the complete network, i.e., $\boldsymbol{Q}_{ij} = \frac{1}{n-1}$ for all $j \neq i$, the information about the troubled bank has no value to creditors since the defaulting bank will carry a $\frac{1}{n-1}$ share of their interbank loans, no matter where it is located in the network. However, when banks are linked in a ring network, i.e., $\boldsymbol{Q}_{i,i+1} = \boldsymbol{Q}_{n,1} = 1$, information regarding the defaulting bank's position becomes critical to creditors. The creditors associated with bank 2, which is the sole creditor of the distressed bank, will be very aggressive in withdrawing early since they understand that bank 2 is facing a large counterparty risk from the collapse of bank 1. I formalize this argument in Proposition 10.

Proposition 10. When the information about the initially distressed bank is available, there exists an $n_0 > 0$ such that the ring network consisting of $n < n_0$ banks is more fragile than the complete network of n banks (when the total interbank lending $\boldsymbol{y} < y_0\mathbf{1}$).

This result is similar to one found by Allen and Gale (2000). When the source of the contagion is public information to all market participants, networks with more diversified connections have the advantage of diversifying the shocks and thus mitigating the contagion. However, the network structure could influence the transmission of the initial collapse; more importantly, it could affect the strategic decisions of the creditors. The collapse of the first bank will generate serious panic among creditors from nearby banks in financial networks with less diversified connections. The panic on any single bank will have negative externality on the liquidity of its creditor banks since interbank loans are less likely to be repaid if the bank is being bombarded by creditors' demand. This, in turn, will trigger additional demand on those banks. This contagious effect will be easily passed to the next group of creditors whose banks have heavily weighted connections with the neighbors of the defaulting bank. The effect is significantly weakened when banks have very lightly weighted connections in a complete network. However, if there are a large number of banks in a ring network, the contagious effect would be relatively small for banks that are sufficiently far from the initial distressed bank. By contrast, all banks in a complete network would be equally affected.

From the above discussion, it is evident that, when there is a distressed bank in the network before creditors making their withdrawal decisions, the stability of ring networks will be very sensitive to creditor's beliefs and information about the location of the initial distressed bank. The complete network is not sensitive to these beliefs and information. Whether information could help to deter creditors panicking depends on the beliefs they hold in the absence of this information. In Proposition 11, I posit that information disclosure is rarely desirable for neutral creditors.

Proposition 11. Consider the ring network. When creditors hold neutral beliefs in the absence of information about the distressed bank, not disclosing information about the source of contagion is welfare improving if the size of network $n \leq n_1$ ($n_1 \geq n_0$).

This outcome is intuitive. If all creditors have perfect information about the financial linkages with the distressed bank, the sole creditor bank of the distressed bank will experience aggressive demand from outside creditors because this bank is sure to receive nothing from the interbank market due to the default of its counterparty. The less diversified connections in the ring network are efficient in propagating one bank's collapse to the next, thereby making the "domino" effect permissible. Under information disclosure, the contagious panic from the initial distressed bank facilitates financial contagion. Thus, when all banks are not very far from the distressed bank, i.e., the size of network is relatively small, information disclosure will make each bank more fragile. If creditors are extremely cautious, then more information about the distressed bank could be helpful in restoring confidence in the financial network.

These exercises demonstrate the desirability of the ex-ante commitment to information disclosure. The Federal REserve Board in the United States and the central bank in Europe conduct periodic stress tests for large banks and disclose information about their ability to sustain future shocks. The question of whether this mandatory disclosure could help to build stability of the financial system has been investigated by several authors (e.g., Bouvard et al. (2015), Goldstein and Sapra (2199), Goldstein and Leitner (2015) and Castro et al. (2015)). For example, Alvarez and

Barlevy (2014) find that voluntary or mandatory information disclosure could reduce the uncertainty of outside investors and make the financial system more robust. The impact of information disclosure depends on the network structure and the cautiousness of creditors, among other things. In my model, when there is a distressed bank in the network, information about this distressed bank has no value in a perfectly diversified network. From the perspective of panic-based bank runs, I find that information disclosure could trigger contagious panic from the neighboring banks of the initial distressed bank and contribute to the fragility of the financial network. Information disclosure is desirable only when creditors are cautious and always assume the worst when information is not available, and/or a large number of banks are interconnected in an undiversified network.

Cascade of Defaults

In the previous discussion, I considered the case in which all interbank lending is relatively low, i.e., $\boldsymbol{y} < y_0\mathbf{1}$. Under this assumption, I constructed the unique equilibrium in which creditors take a threshold strategy in their withdrawal decisions. However, when interbank lending is relatively high (or banks are facing relatively high counterparty risks), multiple equilibria could arise. One such equilibrium could occur when all creditors withdraw funds early, independent of their private information. I call this strategy *complete panic*. In this *worst* possible equilibrium, banks become insolvent, independent of the realization of liquidity shocks. When complete panic takes place, costly early liquidation will occur with probability 1 and thus the economy is extremely inefficient. By investigating the worst possible equilibrium in the default case, I show how different financial networks could make creditors panic completely and thus facilitate the cascade of defaults in the system.[32]

When there is a distressed bank in the financial network and this is public information to all creditors, the worst possible equilibrium can exist because there is no upper dominance region for the strategies of creditors. For example, the creditors of the neighboring bank of the distressed bank in a ring network understand that this distressed bank cannot pay anything to their banks. If all other creditors run on this bank($w = 1$), the bank's residual liquidity for interbank loan repayment is $\theta - 1$ is lower than the amount of interbank loans y, regardless of the realization of θ. Thus, an equilibrium could occur in which all creditors ignore their private information and panick

[32] Considering the planner or regulator is a *max min* agent, which maximizes the expected utility focusing on the worst possible equilibrium provides the rationalization for focusing on the worst possible equilibrium.

completely. This is a rational strategy for creditors and the bank will default, regardless of the strength of its liquidity position. Let m denote the number of banks experience complete panicking in a financial network. In the following proposition, I examine the case where creditors ignore their private information and coordinate on withdrawing even if the liquidity realization of all banks is at its maximum, or $\bar{\theta}$. Proposition 12 shows how m depends on the level of interbank lending y under different network structures.

Proposition 12. The number m of banks experiencing complete panic depends on the interbank lending y and the network structure. Consider the ring and complete networks with $y > \bar{\theta} - 1$.

1. In the ring network, $m[r] = [\frac{y}{\bar{\theta}-1}]$.
2. In the complete network, $m[c] = n \times \mathbb{1}[y > (n-1)(\bar{\theta} - 1)]$.
3. When $y \leq (n-1)(\bar{\theta} - 1)$, $m[c] = 0 < m[r]$.

In Proposition 12, I posit that the cascade of defaults is more permissible in a ring network. The adverse shock from the initial distressed bank is determined by the size y of the interbank bank loans. With larger values of y, the banks connected to the distressed bank will face higher counterparty risk. In a ring network, the adverse effect from the initial distressed bank cannot be absorbed by all banks as it is in a complete network. Creditors who understand this become very responsive to the transmission of the initial shock and a relatively small y can make the cascade of defaults possible. By contrast, in a complete network, each bank is at a "symmetric" position to the initial distressed bank. Thus, either all banks or no bank will face complete panic. The initial default of the troubled bank will have a smaller effect on each bank's balance sheet because the connections are perfectly diversified. In the ring network, banks farther from the initial distressed bank are more difficult to affect. Thus, the cascade of defaults will be able to reach the farthest bank only when the interbank lending y is sufficiently high.

4 Discussion

In this section, I discuss the robustness of my results by considering the possibility of their extension to incorporate alternative assumptions about seniority and the liquidation value of the long-term investments. I discuss an important limitation of my model, that the formation of the financial network is endogenous, and show how my model can shed light on the network formation problem for financial institutions.

Partial Liquidation: $\lambda > 0$

Up to now, I have assumed that the long-term investment cannot be pledgable and that it cannot be liquidated in the intermediate period, that is, $\lambda = 0$. Here, I consider the case where long-term projects can be liquidated, but at a discounted price.

Suppose that banks can sell and recover their share of the riskless return from the long-term investment, i.e., $\lambda \in (0, 1)$. This ratio is uniform for all banks. In this context, when bank i can meet the demand from creditors but cannot fully repay the interbank loans, i.e., $\theta_i + \sum_{k \neq i} x_{ik} < w_i + y \leq \theta_i + \lambda A + \sum_{k \neq i} x_{ik}$,[33] the bank will liquidate a $\frac{w-\theta}{\lambda A}$ share of the long-term investment and use the proceeds from this liquidation to pay early withdrawers and remain solvent. Moreover, if bank i cannot fully repay the interbank loans even if it liquidates all long-term investments, i.e., $w_i > \theta_i + \lambda A + \sum_{k \neq i} x_{ik}$, it defaults.

I further relax the assumption that $A > 1 + r$ by allowing the long-term return A to be any positive number. Note that when the value of the long-term investment is not guaranteed to be high enough to repay all creditors who did not withdraw early, rolling over is more risky since the remaining creditors may receive a payment lower than $1 + r$. [34] The payoff for patient creditors is given by

$$u(a_i = 0, \theta, w) = \begin{cases} 1 & if\ \theta_i + \lambda A + \sum_{k \neq i} x_{ik} \geq w_i + y_i \\ \min\left\{1, \frac{1+\theta_i+\lambda A+\sum_{k \neq i} x_{ik}}{1+w_i}\right\} & \text{otherwise} \end{cases}$$

and

$$u(a_i = 1, \theta, w) = \begin{cases} \min\left\{1 + r, \ \frac{(1-\frac{w_i-\theta_i}{\lambda A})A}{1-w_i}\right\} & if\ \theta_i + \lambda A + \sum_{k \neq i} x_{ik} \geq w_i + y_i \\ 0 & \text{otherwise.} \end{cases}$$

[33] Here, I assume that $A_i = A$ to maintain the symmetry.

[34] Note that in this case, a bank run may occur at $t = 2$ if $\frac{(1-\frac{w_i-\theta_i}{\lambda A})A}{1-w_i} < 1 + r$. However, it is easy to see that the probability of a bank run still depends on creditors' strategic withdrawal decisions, because higher values of w_i will decrease the residual liquidity $(1 - \frac{w_i-\theta_i}{\lambda A})A$ and make the condition of default at $t = 2$ more likely to be satisfied.

These assumptions strengthen the strategic complementarity in the non-default regime, since creditors will have stronger incentives to roll over if fewer creditors withdraw in that regime, i.e., $\min\left\{1+r,\ \frac{(1-\frac{w_i-\theta_i}{\lambda A})A}{1-w_i}\right\}$ is decreasing in w_i. However, the lack of strategic complementarity is still present in the default regime and the clearing payment vector satisfies the same condition as in Equation (2). Hence, the main results will still be valid under this set of assumptions.[35] Moreover, the impact of diversifying connections in increasing the ex-ante probability of default (Proposition 3) will be even stronger. Apart from the intuition provided in the Proposition 3 that the shift of distribution of interbank repayment will provide extra incentive for creditors to run in the default regime, the shift of probability distribution in the non-default regime under this setting will also incentivize creditors to run, since creditors' payoff of not running is lower when their bank lower interbank repayment from its debtor banks in the non-default regime.

Equal Seniority

In the main part of this paper, I have assumed that deposits or short-term debts are senior to unsecured interbank loans. By considering the case in which these have equal seniority, I show that this assumption is not essential for the driving mechanism of my results.

Suppose that deposits or short-term debts have equal seniority to interbank loans. In the case of a default, the creditors of bank i who withdraw early will split the liquid assets with the banks who have interbank claims on bank i. The payoffs for the creditors are given by:

$$u(a_i=0,\theta,w)=\begin{cases}1 & if\ \theta_i+\sum_{k\neq i}x_{ik}\geq w_i+y_i\\ \frac{1+\theta_i+\sum_{k\neq i}x_{ik}}{1+w_i+y_i} & \text{otherwise}\end{cases}$$
$$u(a_i=1,\theta,w)=\begin{cases}1+r & if\ \theta_i+\sum_{k\neq i}x_{ik}\geq w_i+y_i\\ 0 & \text{otherwise.}\end{cases} \tag{5}$$

The interbank repayment will be $x_{ij}=\min\{y_{ij},y_{ij}\frac{\theta_i+\sum_{k\neq i}x_{ki}+1}{w_i+y_i+1}\}$. Thus, the clearing payment vector

[35] Note that the complication in the proof of the existence and uniqueness of the equilibrium in a global game setting comes from the lack of global strategic complementarity as in Goldstein and Pauzner (2005). The assumption strengthening the strategic complementarity will not change the proof of this result.

$\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{\theta})$ will satisfy the following condition,

$$\Gamma(\boldsymbol{x}; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{\theta}) = \min\{\boldsymbol{y}, \boldsymbol{W}(\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{\theta} + \mathbf{1})\},$$

where $\boldsymbol{W}$ is an $n \times n$ diagonal matrix with $\boldsymbol{W}_{ii} = \frac{y_i}{1+w_i+y_i}$.

It is easy to verify that the fixed point of Γ is generically unique since $\boldsymbol{W}_{ii} < 1$ and the mapping Γ is weakly increasing and concave in $\boldsymbol{x}$. Moreover, Γ has similar properties to the mapping Φ defined in Equation (2) (See Lemmas 4 and 5 in the Appendix). Furthermore, in the case of equal seniority, the payoff in Equation (5) has the same features of strategic complementarity as the payoff in the case of absolute seniority (Equation 3). For example, in the default regime, creditors will have lower incentives to withdraw early when the interbank repayment $\sum_{k \neq i} x_{ik}$ is lower. Hence, my results are robust to the assumption of equal seniority.

Network Formation

In this paper, the interbank network is assumed to be exogenously given and the benefits from building financial linkages are not modeled. However, the above discussion of financial fragility and contagion leads me to examine the potential externalities of the network formation problem.

The panics among creditors that incentivize them to withdraw their funds early is an important source of liquidity risk. From the perspective of panic-based bank runs, by building financial connections, banks not only make themselves more fragile, but also contribute to the fragility of other connected banks because of the contagious nature of panics. Banks will only take their own risks into consideration when building bilateral financial linkages and the counterparty's risk might be contractable as in Acemoglu et al. (2015a). However, contracts cannot be made contingent on the risk of the counterparty's counterparty, and, in general, financial institutions will ignore the contribution of building financial connections to the systemic risk. Based on my analysis, if considering the time 0 problem when banks are deciding to build financial connections with other banks, I would conjecture that banks will borrow (or lend) inefficiently high amounts from (or to) other banks and the pattern of financial linkages will be inefficiently over-diversified. Both high exposure and over-diversification of financial connections will make the financial system more fragile.

5 Conclusion

This paper provides a framework to examine the fragility of financial networks from the perspective of panic-based bank runs. Under this framework I developed in the paper, I study how financial fragility depends on the structure of interbank networks and the policy of information disclosure. The conventional wisdom about the systemic risk and financial networks–"Robust yet fragile"– is that the interconnected financial system is robust during good times, but could be fragile when some adverse shock hits. In this paper, I further examine this argument by comparing different network structures. The main result of the paper is to show that more diversified network structures are more fragile during normal times, while they can be more robust when the financial system is facing a financial contagion. The less diversified network structure is robust during normal times, but could be very sensitive to the availability of information and the beliefs held by creditors when a crisis is underway, and thus could be very fragile during bad times. Hence, I provide a unique way of examining the relation between systemic risk and network structure by incorporating the strategic decisions of short-term creditors into the model.

By incorporating creditors' panics into the analysis of financial stability and network structure, I provide some insights into the effective design of relevant policies for financial markets, e.g., stress tests and policies for information disclosure. I find that disclosing more information about a bank's counterparty exposure or potential losses is rarely desirable because it may trigger panics from the banks that are tightly connected to the initial distressed bank and the contagious nature of panics will facilitate financial contagion across the network. More information only helps to reduce ambiguity and restore confidence if creditors are cautious and always assume the worst when information about the distressed banks is not available.

Studying the systemic risk from the perspective of panic-based bank runs raises a number of interesting questions. In my model, I assume that the financial network is already established, and so I have not modeled the endogenous network formation. It would be worthwhile to investigate the network formation problem, in which each bank takes the liquidity risk originating from the endogenous panics among its creditors, and from the panic among creditors of linked banks, into consideration when building financial connections. Another interesting question, which has not been investigated in the literature, is the impact of the policy of information disclosure on the strategic formation of financial networks. From this perspective, information disclosure could have a more complicated impact on financial fragility. For instance, in this paper I suggest the possibility that, under a policy of mandatory disclosure, financial institutions will tend to form more diversified connections to reduce their sensitivity to information when a financial crisis hits. However, the resulting network with diversified connections would endogenously increase the likelihood of the first bank run. A more complete model that also examines network formation is needed to fully answer these important questions. I leave this for future research.

Appendix

A The Clearing Payment Vector

Preliminaries

Let $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, and define lattice operators $\vee, \wedge$ as follows:

$$\boldsymbol{x} \wedge \boldsymbol{y} = (\min\{x_1, y_1\}, \min\{x_2, y_2\}, \dots, \min\{x_n, y_n\}),$$

and

$$\boldsymbol{x} \vee \boldsymbol{y} = (\max\{x_1, y_1\}, \max\{x_2, y_2\}, \dots, \max\{x_n, y_n\}).$$

For $\boldsymbol{x} \in \mathbb{R}^n$, let the lattice element $\boldsymbol{x}^+$ be defined by

$$\boldsymbol{x}^+ = (\max\{x_1, 0\}, \max\{x_2, 0\}, \dots, \max\{x_n, 0\}).$$

A pointwise ordering $\leq$ is defined for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ by

$$\boldsymbol{x} \leq \boldsymbol{y} \iff x_i \leq y_i \text{ for all } i \in \{1, 2, \dots, n\}.$$

Let $\mathbf{1}$ denote the $n-$dimensional vector whose components are all equal to 1, i.e., $\mathbf{1} = (1, 1, \dots, 1)^T$, and let $\mathbf{0}$ denote the zero vector, i.e., $\mathbf{0} = (0, 0, \dots, 0)^T$.

Clearing Payment Vector as a Fixed Point

Let $\Phi : \prod_{i=1}^n [0, y_i] \to \prod_{i=1}^n [0, y_i]$ be the mapping defined by $\Phi(\boldsymbol{x}; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) = [\min\{\boldsymbol{e} + \boldsymbol{Q}\boldsymbol{x}, \boldsymbol{y}\}]^+$. Then Φ is a one-to-one mapping and the clearing payment vector $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$ is a fixed point of Φ, i.e., $\Phi(\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}); \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) = \boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$. We can see from Proposition 1 of Acemoglu et al. (2015b), Theorem 1 of Eisenberg and Noe (2001), and Lemma 1 that the clearing payment vector $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$ is unique when $\sum_{i=1}^n e_i \neq 0$.

Lemma 4. Let $(\boldsymbol{Q}, \boldsymbol{y})$ be a network, and let $\boldsymbol{e}$ satisfy $\sum_{i=1}^n e_i \neq 0$. Then for all $\boldsymbol{x}_0 \in \prod_{i=1}^n [0, y_i]$, the sequence $\{\boldsymbol{x}_{i+1} = \Phi(\boldsymbol{x}_i; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})\}_{i=0}^\infty$ will converge to the fixed point $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$.

Proof. Now, Φ is weakly increasing in $\boldsymbol{x}$, and so, if $\boldsymbol{x}_0 \geq \Phi(\boldsymbol{x}_0; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$, then $\boldsymbol{x}_{i+1} \geq \boldsymbol{x}_i$ for all

i. It follows that $\boldsymbol{x}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}) \geq \boldsymbol{x}_0$. A similar argument shows that if $\boldsymbol{x}_0 \leq \Phi(\boldsymbol{x}_0;\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e})$, then $\boldsymbol{x}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}) \leq \boldsymbol{x}_0$. ☐

Convex Combinations and the Clearing Payment Vector

Suppose that the financial network $(\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma)$ is a $\gamma-$convex combination of two different financial networks, $(\boldsymbol{Q}_1,\boldsymbol{y}_1)$ and $(\boldsymbol{Q}_2,\boldsymbol{y}_2)$, i.e., $\boldsymbol{Q}_\gamma = \gamma\boldsymbol{Q}_1 + (1-\gamma)\boldsymbol{Q}_2,\boldsymbol{y}_\gamma = \gamma\boldsymbol{y}_1 + (1-\gamma)\boldsymbol{y}_2$. Further assume that $\boldsymbol{e}_\gamma = \gamma\boldsymbol{e}_1 + (1-\gamma)\boldsymbol{e}_2$.

Lemma 5. The clearing payment vector $\boldsymbol{x}(\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma)$ satisfies the following inequalities:

$$\boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \wedge \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2) \leq \boldsymbol{x}(\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma) \leq \boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \vee \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2).$$

Proof. Define $W^+(\boldsymbol{x}) = \Phi(\boldsymbol{x};\boldsymbol{Q},\boldsymbol{y}_1,\boldsymbol{e}_1) \vee \Phi(\boldsymbol{x};\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2)$ and $W^-(\boldsymbol{x}) = \Phi(\boldsymbol{x};\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \wedge \Phi(\boldsymbol{x};\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2)$. Since $\Phi(\boldsymbol{x};\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma) = \left[\min\{\boldsymbol{e}_\gamma + \boldsymbol{Q}_\gamma\boldsymbol{x},\boldsymbol{y}_\gamma\}\right]^+$ is linear and monotone in γ , we see that

$$W^-(\boldsymbol{x}) \leq \Phi(\boldsymbol{x};\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma) \leq W^+(\boldsymbol{x}). \tag{A.1}$$

Now, W^+ and W^- are monotone maps defined on $\prod_{i=1}^n[0,y_i]$ with fixed points in this interval. Suppose that $\boldsymbol{x}^+$ and $\boldsymbol{x}^-$ are fixed points of W^+ and W^-, respectively. It follows from equation (A.1) that

$$\boldsymbol{x}^- \leq \boldsymbol{x}(\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma) \leq \boldsymbol{x}^+.$$

Since $\boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \wedge \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2)$ is a sub-solution to W^- and $\boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \vee \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2)$ is a super-solution to W^+, the inequality

$$\boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \wedge \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2) \leq \boldsymbol{x}^- \leq \boldsymbol{x}(\boldsymbol{Q}_\gamma,\boldsymbol{y}_\gamma,\boldsymbol{e}_\gamma) \leq \boldsymbol{x}^+ \leq \boldsymbol{x}(\boldsymbol{Q}_1,\boldsymbol{y}_1,\boldsymbol{e}_1) \vee \boldsymbol{x}(\boldsymbol{Q}_2,\boldsymbol{y}_2,\boldsymbol{e}_2)$$

holds, as required. ☐

B Proofs for Regular and Symmetric Networks

Proof of Lemma 1: The proof of the existence and uniqueness of the fixed point vector $\boldsymbol{x}$ follows the proof of Proposition 1 of Acemoglu et al. (2015b).

Note that $e_i = \theta_i - w_i = \theta_i - Pr(s_i \in \mathbb{S}_i|\theta)$ is a function of θ_i. The fixed point $\boldsymbol{x}$ of e_i is unique unless $\sum_{i=1}^n e_i = \sum_{i=1}^n (\theta_i - w_i) = 0$. For each i, since θ_i and e_i are both continuously distributed, $\sum_{i=1}^n e_i = 0$ only occurs in a non-generic set $\{\boldsymbol{\theta}|\sum_{i=1}^n e_i = 0\}$. Thus, the clearing payment vector $\boldsymbol{x}$ exists and is generically unique. □

Proof of Lemma 2: First, I show that, given any possible strategy portfolio of other banks' creditors $\{\mathbb{S}_j\}_{j\neq i}$, the iterated elimination of dominated strategies for bank i's patient creditors can be performed. This will be achieved by showing that the iterated elimination of dominated strategies can be performed from both sides, i.e., the upper dominance region and the lower dominance region.

Suppose that the worst possible scenario for creditors' withdrawal decisions occurs. In this case, $\mathbb{S}_i = \bar{\mathbb{S}} \equiv [\underline{s}, \bar{s}]$, i.e., all creditors will withdraw their funds, independent of their private information regarding θ_i. In this context, $w_i = 1$ and $e_i = \theta_i - 1$. Next, consider the worst possible scenario for the repayment of interbank loans, where bank i receives nothing from its debtor banks, i.e., $\sum_{j\neq i} x_{ij} = 0$. In this context, the bank's liquidity is $1 + \theta_i$ while its total obligation is $1 + 1 + y$. A creditor who receives information $s_i \in [1 + y + \frac{\sigma}{2}, \bar{s}] \subset \bar{\mathbb{S}}$ understands that the realization of θ_i is not lower than $1 + y$. Even in the worst possible case, a bank with $\theta_i \geq 1 + y$ will be able to fulfill its obligations at $t = 1$. Hence, those creditors with private information $s_i \geq 1 + y + \frac{\sigma}{2}$ will never withdraw early. This gives rise to the upper dominance region.

Now suppose that the best possible scenario for creditors' withdrawal decisions occurs. In this case, $\mathbb{S}_i = \Phi$, i.e., no creditors will withdraw early, regardless of the private information that they receive. A creditor who receives information $s_i \in [\underline{s}, -\frac{\sigma}{2}) \subset \bar{\mathbb{S}}$ understands that $\theta_i < 0$. The liquid assets held by bank i are of value $1 + \theta_i + \sum_{j\neq i} x_{ij}$, which is less than the bank's total outstanding liability $1 + y$. Thus, creditors with private information $s_i \in [\underline{s}, -\frac{\sigma}{2}) \subset \bar{\mathbb{S}}$ will never delay their withdrawals even if all other creditors will do so. This gives rise to the lower dominance region. It is assumed that creditors will update their beliefs about the strategies of other creditors and adjust the "best possible scenario" and "worst possible scenario" accordingly.

By an iterated elimination of dominated strategies, we can see that there are two thresholds, $\bar{s}_i(\{\mathbb{S}_j\}_{j\neq i})$ and $\underline{s}_i(\{\mathbb{S}_j\}_{j\neq i})$ such that the only rationalizable action for any patient creditor of bank i is to never withdraw early if $s_i \geq \bar{s}_i(\{\mathbb{S}_j\}_{j\neq i})$, and to never delay withdrawal if $s_i < \underline{s}_i(\{\mathbb{S}_j\}_{j\neq i})$. Thus, the worst possible equilibrium could be that all creditors withdraw early whenever $s_i < \bar{s}_i(\{\mathbb{S}_j\}_{j\neq i})$. The best possible equilibrium could occur when creditors choose not to withdraw early if and only if $s_i \geq \underline{s}_i(\{\mathbb{S}_j\}_{j\neq i})$.

If $\bar{s}_i(\{\mathbb{S}_j\}_{j\neq i}) > \underline{s}_i(\{\mathbb{S}_j\}_{j\neq i})$, then there are at least two monotone equilibria and there could also be non-monotone equilibria. The following argument shows that there is a unique monotone equilibrium, i.e., $\bar{s}_i(\{\mathbb{S}_j\}_{j\neq i}) = \underline{s}_i(\{\mathbb{S}_j\}_{j\neq i})$, which is actually the unique equilibrium for a given $\{\mathbb{S}_j\}_{j\neq i}$. In a monotone equilibrium, the creditors play the same strategy. Let $s_i^\star(\{\mathbb{S}_j\}_{j\neq i}) \in [\underline{s}, \bar{s}]$ be any possible threshold. If $m \in [0, 1]$ is a creditor of bank i, then an equilibrium occurs if m uses the same strategy as all of the other creditors who withdraw early if and only if $s_i < s_i^\star(\{\mathbb{S}_j\}_{j\neq i})$. Suppose that the equilibrium is calculated in this way. I now show that there is a unique solution to the equation $s_i^\star(\{\mathbb{S}_j\}_{j\neq i})$.

First, the expected payoff difference $H(s_{im}, s_i^\star, \{\mathbb{S}_j\}_{j\neq i})$ (between delaying withdrawal and withdrawing early) for each creditor m of bank i, given the other banks creditors' strategies $\{\mathbb{S}_j\}_{j\neq i}$ and information s_{im} must be calculated. We see from equation (4) that

$$H(s_{im}, s_i^\star, \{\mathbb{S}_j\}_{j\neq i}) = \int_{\boldsymbol{\theta}_{-i}} V(s_{im}, s_i^\star) dJ(\boldsymbol{\theta}_{-i}|s_{im}),$$

where $\boldsymbol{\theta}_{-i}$ is the realization of $\{\theta_j\}_{j\neq i}$ and $J(\boldsymbol{\theta}_{-i}|s_{im})$ denotes the CDF of $\boldsymbol{\theta}_{-i}$, given s_i. Moreover, $V(s_{im,} s_i^\star)$ is the payoff difference (between delaying withdrawal and withdrawing early) for a creditor with information s_{im}, given the realization of $\boldsymbol{\theta}_{-i}$, the strategies of the other banks' creditors $\{\mathbb{S}_j\}_{j\neq i}$, and the strategy $s_i^\star$ of the other creditors in bank i. In the above equation, $V(s_{im}, s_i^\star)$ is calculated as follows:

$$\begin{aligned} V(s_{im}, s_i^\star) &= \int_{\{\theta_i|x_i=y\}} r dF(\theta_i|s_{im}) + \int_{\{\theta_i|0<x_i<y\}} (-1) dF(\theta_i|s_{im}) \\ &+ \int_{\{\theta_i|x_i=0\}} -\frac{\theta_i + \sum_{j\neq i} x_{ij}(\theta_i) + 1}{w_i + 1} dF(\theta_i|s_{im}). \end{aligned}$$

Note that, given $\boldsymbol{\theta}_{-i}$, $\{\mathbb{S}_j\}_{j\neq i}$ and $s_i^\star$, the value of $\sum_{j\neq i} x_{ij}$ depends on how much bank i can pay to its creditor banks and, thus, it is a function of θ_i. If $s_i^\star$ is an equilibrium for $\{\mathbb{S}_j\}_{j\neq i}$, then $H(s_i^\star, s_i^\star, \{\mathbb{S}_j\}_{j\neq i}) = \int_{\boldsymbol{\theta}_{-i}} V(s_i^\star, s_i^\star) dJ(\boldsymbol{\theta}_{-i}|s_i) = 0$. The threshold liquidity $\theta_i^\star(\boldsymbol{\theta}_{-i}, s_i^\star, \{\mathbb{S}_j\}_{j\neq i})$ is defined to be the threshold at which bank i can exactly fulfill its obligations. An additional threshold liquidity, $\theta_i^0(\boldsymbol{\theta}_{-i}, s_i^\star, \{\mathbb{S}_j\}_{j\neq i})$ is defined to be the threshold at which bank i can exactly fulfill all liabilities, or exactly pay the withdrawals from its creditors. The detailed relationship between these variables is given by:

$$\theta_i^\star + \sum_{j \neq i} x_{ij}(\theta_i^\star) = w_i(\theta_i^\star) + y, \text{ and } \theta_i^0 + \sum_{j \neq i} x_{ij}(\theta_i^0) = w_i(\theta_i^0).$$

The next step is to write the expression for V in terms of w_i. Now, w_i is uniformly distributed for a threshold creditor with information $s_i^\star$ since

$$Pr(w_i < k | s_i^\star) = Pr(F(\frac{s_i^\star - \theta_i}{\sigma}) < k | s_i = s_i^\star) = k.$$

Moreover, V can be written as an integral of w_i:

$$V(s_i^\star, s_i^\star) = \int_0^{w_i^\star} r dw_i + \int_{w_i^\star}^{w_i^0} (-1) dw_i + \int_{w_i^0}^1 \left(-\frac{\theta_i + \sum_{j \neq i} x_{ij}(w_i) + 1}{w_i + 1} \right) d\theta_i, \tag{B.1}$$

where $w_i^\star$ and w_i^0 are the aggregate withdrawals when $\theta_i = \theta_i^\star$ and $\theta_i = \theta_i^0$, respectively. In particular,

$$w_i^\star(\theta_{-i}, \mathbb{S}) = \theta_i + \sum_{j \neq i} x_{ij} - y = \frac{1}{1+\sigma} \left(s_i^\star + \frac{1}{2}\sigma + \sum_j x_{ij}(w_i^\star) - y \right),$$

and

$$w_i^0(\theta_{-i}, \mathbb{S}) = \theta_i + \sum_{j \neq i} x_{ij} = \frac{1}{1+\sigma} \left(s_i^\star + \frac{1}{2}\sigma + \sum_{j \neq i} x_{ij}(w_i^0) \right).$$

Simplifying equation (B.1) yields

$$\begin{aligned} V(s_i^\star, s_i^\star) &= r w_i^\star - (w_i^0 - w_i^\star) + \int_{w_i^0}^1 \left(-\frac{\theta_i + \sum_{j \neq i} x_{ij}(w_i^0) + 1}{w_i + 1} \right) d\theta_i \\ &= \frac{1+r}{1+\sigma} \left(\sum_j x_{ij}(w_i^\star) - \sum_{j \neq i} x_{ij}(w_i^0) - y \right) + \frac{r-\sigma}{1+\sigma} \left(s_i^\star + \frac{1}{2}\sigma + \sum_{j \neq i} x_{ij}(w_i^0) \right) \\ &\quad - \left(s_i^\star + \frac{1}{2}\sigma + \sum_{j \neq i} x_{ij}(w_i^0) + 1 + \sigma \right) \left[\ln 2 - \ln \left(\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j \neq i} x_{ij}(w_i^0)}{1+\sigma} + 1 \right) \right]. \end{aligned}$$

Note that, for any given $\{\theta_j\}_{j \neq i}$ and $\{\mathbb{S}_j\}_{j \neq i}$, the equality $\sum_{j \neq i} x_{ij}(w_i) = \sum_{j \neq i} x_{ij}(w_i^0)$ is satisfied for all $w_i > w_i^0$. The reason for this equality is that when $w_i > w_i^0$, bank i defaults with $x_i = 0$. If $\boldsymbol{x}$ is a clearing payment vector with respect to $(e_1, e_2, \ldots, e_i, \ldots, e_n)$, then any further decreases in θ_i (or further increases in w_i) will only change e_i and $\boldsymbol{x}$ will still be an clearing vector for the interbank market. The payoff difference, $V(s_i^\star, s_i^\star)$ can be rewritten in the form

$$V(s_i^\star, s_i^\star) = \frac{1+r}{1+\sigma}\left(\left(\sum_j x_{ij}(w_i^\star) - \sum_{j\neq i} x_{ij} w_i^0) - y\right) + U(s_i^\star, s_i^\star)\right),$$

where

$$\begin{aligned} U\left(s_i^\star, s_i^\star, \sum_{j\neq i} x_{ij}(w_i^0)\right) :&= \frac{r-\sigma}{1+\sigma}\left(s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)\right) - \left(s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0) + 1 + \sigma\right) \\ &\times \left[\ln 2 - \ln\left(\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)}{1+\sigma} + 1\right)\right]. \end{aligned} \tag{B.2}$$

The value of the expected payoff difference $H(s_{im}, s_i^\star, \{\mathbb{S}_j\}_{j\neq i})$ for each creditor m of bank i, given the other banks creditors' strategies $\{\mathbb{S}_j\}_{j\neq i}$ and information s_{im} can now be found:

$$H(s_i^\star, s_i^\star, \{\mathbb{S}_j\}_{j\neq i}) = \int_{\boldsymbol{\theta}_{-i}} \frac{1+r}{1+\sigma}\left(\sum_j x_{ij}(w_i^\star) - \sum_{j\neq i} x_{ij}(w_i^0) - y\right) + U(s_i^\star, s_i^\star) dJ(\boldsymbol{\theta}_{-i}|s).$$

The following argument shows that the term $\int_{\boldsymbol{\theta}_{-i}} \frac{1+r}{1+\sigma}(\sum_j x_{ij}(w_i^\star) - \sum_{j\neq i} x_{ij}(w_i^0) - y) dJ(\boldsymbol{\theta}_{-i}|s)$ in the above expression is a constant which only depends on the financial network $(\boldsymbol{Q}, \boldsymbol{y})$. Note that, given the liquidity $\{\theta_j\}_{j\neq i}$ and withdrawal strategies $\{\mathbb{S}_j\}_{j\neq i}$, when bank i receives $\sum_j x_{ij}(w_i^\star)$ under the withdrawal strategy $s_i^\star$, bank i can meet its obligations and pay exactly y to each of its creditor banks. Similarly, when bank i receives $\sum_j x_{ij}(w_i^0)$, bank i defaults on its interbank repayments and pays exactly 0 to each of its creditor banks. For any realization of $\theta_{j(j\neq i)}$,

$$\begin{aligned} e_j &= \theta_j - w_j(\theta_j) = \theta_j - F\left(\frac{s_j^\star - \theta_j}{\sigma}\right) \\ &= \left(1+\frac{1}{\sigma}\right)\theta_j - \frac{s_j^\star + \frac{\sigma}{2}}{\sigma} \in \left[\left(1+\frac{1}{\sigma}\right)\underline{\theta} - \frac{s_j^\star + \frac{\sigma}{2}}{\sigma}, \left(1+\frac{1}{\sigma}\right)\bar{\theta} - \frac{s_j^\star + \frac{\sigma}{2}}{\sigma}\right]. \end{aligned}$$

Consider any realization of $\{\theta_j\}_{j\neq i}$ and any vector of residual liabilities, $\{e_j\}_{j\neq i}$. We can see that $\sum_{j\neq i} x_{ij}(w_i^\star)$ (with respect to $\{\theta_j\}_{j\neq i}$) is the same as $\sum_{j\neq i} x_{ij}(w_i^0)$ (with respect to $\{\theta_j + (\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji} y\}_{j\neq i}$).[36] After this adjustment, each bank's residual fundamental e_j remains the same and

[36]The idea is to "reimburse" all banks j the difference between $x_i = 0$ and $x_i = y$ so that the clearing payment vector $\boldsymbol{x}$ remains the same.

the interbank payment for each bank (except bank i) is the same. Hence, [37]

$$\begin{aligned}
&\int_{\boldsymbol{\theta}_{-i}} \tfrac{1+r}{1+\sigma}(\textstyle\sum_j x_{ij}(w_i^\star) - \sum_{j\neq i} x_{ij}(w_i^0))dJ(\boldsymbol{\theta}_{-i}|s) \\
= &\tfrac{1+r}{1+\sigma}\int_{\theta_j\in[\bar{\theta}-(\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y,\bar{\theta}]} \textstyle\sum_j x_{ij}(w_i^\star)dF(\theta_j|s_i) - \tfrac{1+r}{1+\sigma} \displaystyle\int_{\theta_j\in[\underline{\theta},\underline{\theta}+(\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y]} \sum_j x_{ij}(w_i^0)dF(\theta_j|s_i).
\end{aligned}$$

Notice that when $y \leq \min\{-\underline{\theta}, 1-\bar{\theta}\}$, the inequalities $\bar{\theta} - (\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y > 1$ and $\underline{\theta} + (\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y < 0$ hold. Now $x_j = y$ when $\theta_j \in [\bar{\theta} - (\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y, \bar{\theta}]$ for all $j \neq i$, and $x_j = 0$ when $\theta_j \in [\underline{\theta}, \underline{\theta} + (\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y]$ for all $j \neq i$. Thus,

$$\begin{aligned}
\int_{\boldsymbol{\theta}_{-i}} \frac{1+r}{1+\sigma}\left(\sum_j x_{ij}(w_i^\star) - \sum_{j\neq i} x_{ij}(w_i^0) - y\right) dJ(\boldsymbol{\theta}_{-i}|s) &= \frac{1+r}{1+\sigma}y \times \prod_{j\neq i}\frac{(\frac{\sigma}{\sigma+1})\boldsymbol{Q}_{ji}y}{\bar{\theta}-\underline{\theta}} \\
&= \frac{(1+r)\sigma^{n-1}}{(1+\sigma)^n}\frac{y^n}{(\bar{\theta}-\underline{\theta})^n}\prod_{j\neq i}\boldsymbol{Q}_{ji}.
\end{aligned}$$

The payoff indifference is

$$\begin{aligned}
H(s_i^\star, s_i^\star, \{\mathbb{S}_j\}_{j\neq i}) &= \int_{\boldsymbol{\theta}_{-i}} U(s_i^\star, s_i^\star)dF(\boldsymbol{\theta}_{-i}|s_i^\star) \\
&\quad + \frac{(1+r)\sigma^{n-1}}{(1+\sigma)^n}\frac{y^n}{(\bar{\theta}-\underline{\theta})^n}\prod_{j\neq i}\boldsymbol{Q}_{ji} - \frac{1+r}{1+\sigma}y. \qquad \text{(B.3)}
\end{aligned}$$

Notice that $\prod_{j\neq i}\boldsymbol{Q}_{ji} = 0$ whenever there is a $j \neq i$ such that $\boldsymbol{Q}_{ji} = 0$.

In a symmetric and regular network, $\prod_{j\neq i}\boldsymbol{Q}_{ji} > 0$ only if bank i is connected to all other banks. This term is relatively small, even in this case, since

$$\max \frac{(1+r)\sigma^{n-1}}{(1+\sigma)^n}\frac{y^n}{(\bar{\theta}-\underline{\theta})^n}\prod_{j\neq i}\boldsymbol{Q}_{ji} = (\frac{1}{n-1})^{n-1}\frac{(1+r)\sigma^{n-1}}{(1+\sigma)^n}\frac{y^n}{(\bar{\theta}-\underline{\theta})^n} \qquad \text{(B.4)}$$

[38]

[37] For convenience, the notation θ_j is abused and refers to $\boldsymbol{\theta}_{-i}$ in the integral. Essentially, $\int_{\boldsymbol{\theta}_{-i}} d\boldsymbol{\theta}_{-i} = \int_{\theta_1}(\int_{\theta_2}...(\int_{\theta_n} d\theta_n)...d\theta_2)d\theta_1$. The notation $F(.)$ is used to represent the CDF of θ_j and $J(.)$ is used to represent that of $\boldsymbol{\theta}_{-i}$.

[38] Given the financial network $(\boldsymbol{Q}, \boldsymbol{y})$, this constant number is fixed for any $\{\mathbb{S}_j\}_{j\neq i}$. This term will be discussed only when comparing a network to another network which has degree n. There are two ways in which this constant number can be discarded. One way is to assume that $\sigma \to 0$ and the other way is to adopt the improper prior with $\bar{\theta} \to \infty$ and $\underline{\theta} \to -\infty$.

It follows from equation (B.2), that $U(s_i^\star, s_i^\star, \sum_{j\neq i} x_{ij}(w_i^0))$ is increasing in $s_i^\star$ because

$$\frac{dU}{ds_i^\star} = \frac{1+r}{1+\sigma} - \ln \frac{2}{\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)}{1+\sigma} + 1}. \tag{B.5}$$

Notice that $\frac{1}{1+\sigma}\left[s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)\right] = w^0 \in [0,1]$. Although w_i^0 depends on $s_i^\star$, we can see that $w_i^0 > 0$, regardless of the value of $s_i^\star$. When $x_i = 0$, the sum $\sum_{j\neq i} x_{ij}(w_i^0)$ only depends on the other bank's liquidity θ_j, the aggregate withdrawals w_j, and the financial network. We check that

$$\frac{dU}{ds_i^\star} = \frac{r-\sigma}{1+\sigma} + 1 - \ln \frac{2}{\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)}{1+\sigma} + 1} \geq \frac{1+r}{1+\sigma} - \ln 2.$$

It follows that $\frac{dU}{ds_i^\star} \geq 0$ if $\sigma \leq \sigma_0 = \frac{1+r}{\ln 2} - 1$ for any realization of $\{\theta_j\}_{j\neq i}$. Given any $\{\mathbb{S}_j\}_{j\neq i}$, the function $H(s_i^\star, s_i^\star, \{\mathbb{S}_j\}_{j\neq i})$ is increasing in $s_i^\star$ when $\sigma \leq \sigma_0$ and there is a unique solution $s_i^\star \in \bar{\mathbb{S}}$ to the equation $H(s_i^\star, s_i^\star, \{\mathbb{S}_j\}_{j\neq i}) = 0$. Thus, the monotone equilibrium is unique and $\mathbb{S}_i = [\underline{s}, s_i^\star(\{\mathbb{S}_j\}_{j\neq i}))$. □

Proof of Proposition 1: The existence and uniqueness of the symmetric equilibrium will first be established. This will be followed by an argument showing that any equilibrium has to be symmetric.

Given any regular and symmetric financial network $(\boldsymbol{Q}, \boldsymbol{y})$, the existence and uniqueness of a symmetric equilibrium is guaranteed by the fact that $H(s^\star, s^\star, \{\mathbb{S}_j = [\underline{s}, s^\star)\}_{j\neq i})$ is increasing in $s^\star \in \bar{\mathbb{S}}$. Now, suppose that $s_0 = s^\star + \eta$, where $\eta > 0$ is a small constant close to zero. The following argument shows that $H(s_0, s_0, \{\mathbb{S}_j = [\underline{s}, s_0)\}_{j\neq i}) > H(s^\star, s^\star, \{\mathbb{S}_j = [\underline{s}, s^\star)\}_{j\neq i})$.

Consider a bank i and a realization of $(\theta_j)_{j\neq i}$. Then $e_j = \theta_j - F(\frac{s_j^\star - \theta_j}{\sigma}) = \frac{1+\sigma}{\sigma}\theta_j - \frac{s_j^\star + \frac{\sigma}{2}}{\sigma}$ can be written in the form $e_j(s^\star, \theta_j) = e_j(s_0, \theta_j) + \frac{\eta}{\sigma}$. If $\theta_j' = \theta_j + \frac{1}{1+\sigma}\eta$ for all $j \neq i$, then $e_j(s^\star, \theta_j) = e_j(s_0, \theta_j')$. Define $\theta_i^0(s_i^\star, \{\theta_j\}_{j\neq i})$ to be the threshold liquidity such that $x_i = 0$. So, $e_i^0 = \frac{1+\sigma}{\sigma}\theta_i^0 - \frac{s_i^\star + \frac{\sigma}{2}}{\sigma}$. Further, define $\boldsymbol{e} \equiv (e_1(s^\star, \theta_1), e_2(s^\star, \theta_2), \ldots, e_i^0(s^\star, \theta_i^0), \ldots, e_n(s^\star, \theta_n))^T$ and $\boldsymbol{e}' \equiv (e_1(s_0, \theta_1'), e_2(s_0, \theta_2'), \ldots, e_i^0(s_0, \theta_i^0), \ldots, e_n(s_0, \theta_n'))^T$. The equality $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) = \boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}')$ is trivially satisfied. It follows that $\sum_{j\neq i} x_{ij}^0(s^\star, \boldsymbol{e}) = \sum_{j\neq i} x_{ij}^0(s_0, \boldsymbol{e}')$.

Note that,

$$
\begin{aligned}
H(s_0, s_0, \{s_j^\star = s_0\}_{j\neq i}) - H(s^\star, s^\star, \{s_j^\star = s^\star\}_{j\neq i}) &= \int_{\boldsymbol{\theta}_{-i}} \left[U(s_0, s_0) - U(s^\star, s^\star)\right] dJ(\boldsymbol{\theta}_{-i}|s^\star) \\
&= \int_{\theta_j'=\underline{\theta}+\frac{1}{1+\sigma}\eta}^{\bar{\theta}} U\left(s_0, s_0, \sum_{j\neq i} x_{ij}^0(s^0, \theta_j')\right) d(\theta_j') + \int_{\theta_j=\bar{\theta}-\frac{1}{1+\sigma}\eta}^{\bar{\theta}} U\left(s_0, s_0, \sum_{j\neq i} x_{ij}^0(s^0, \theta_j)\right) d(\theta_j) \\
&- \int_{\theta_j=\underline{\theta}}^{\theta_j=\bar{\theta}} U\left(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0(s^\star, \theta_j)\right) d\theta_j \qquad \text{(B.6)} \\
&= \int_{\theta_j=\underline{\theta}+\frac{1}{1+\sigma}\eta}^{\bar{\theta}} \left[U\left(s_0, s_0, \sum_{j\neq i} x_{ij}^0\right) - U\left(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0\right)\right] d\theta_j \qquad \text{(B.7)} \\
&+ \int_{\theta_j=\bar{\theta}-\frac{1}{1+\sigma}\eta}^{\bar{\theta}} U\left(s_0, s_0, \sum_{j\neq i} x_{ij}^0(s^0, \theta_j)\right) d(\theta_j) \\
&- \int_{\theta_j=\underline{\theta}}^{\theta_j=\underline{\theta}+\frac{1}{1+\sigma}\eta} U\left(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0(s^\star, \theta_j)\right) d\theta_j.
\end{aligned}
$$

The function, $U(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0)$, defined in (B.2) is increasing in $s^\star$ and $\sum_{j\neq i} x_{ij}^0$. The first term on the right of equation (B.7) is positive since $s_0 > s^\star$. When η is small and close to zero, $\theta_j \leq \underline{\theta} + \frac{\eta}{1+\sigma} < 0$ and so $e_j = \theta_j - 1 < -1$. It follows that $\sum_{j\neq i} x_{ij}^0 = 0$, since $x_j = 0$ for all j in this case. A similar argument shows that when $\theta_j \geq \bar{\theta} - \frac{\eta}{1+\sigma} > 1$, the equality $\sum_{j\neq i} x_{ij}^0 = y$ is satisfied. Since $s_0 > s^\star$ and $\sum_{j\neq i} x_{ij}^0(s^0, \theta_j) \geq \sum_{j\neq i} x_{ij}^0(s^\star, \theta_j) = 0$, the difference between the second term and third terms is positive. It follows that $H(s^\star, s^\star, \{\mathbb{S}_j = [\underline{s}, s^\star)\}_{j\neq i})$ is increasing in $s^\star$ when $y \leq y_0 = \min\{-\underline{\theta}, \bar{\theta} - 1\}$. Because of the existence of the lower and upper dominance regions as discussed in the proof of Lemma 1, there exists a unique $s^\star(\boldsymbol{Q}, \boldsymbol{y}) \in (\underline{s}, \bar{s})$ such that the equality $H(s^\star, s^\star, \{\mathbb{S}_j = [\underline{s}, s^\star)\}_{j\neq i}) = 0$ is satisfied.

The next step is to show that any equilibrium in a symmetric and regular network must be symmetric. By Lemma 1, any possible equilibrium has the form, $\mathbb{S}_i = [\underline{s}, s_i^\star)$. Suppose that the equilibrium is not symmetric, so there is at least one maximal threshold. After taking a proper permutation from $\mathcal{N}$ to $\mathcal{N}$, one may, without loss of generality, assume that $s_1^\star = \max\{s_1^\star, s_2^\star, \ldots, s_n^\star\}$. Note that $s_i^\star$ denotes the counterparty risk faced by bank i's creditor banks and the expected value of x_j will decrease with the threshold $s_j^\star$. The result holds trivially for the ring network $\boldsymbol{Q}_{i,i+1} = \boldsymbol{Q}_{n,1} = 1$, since if $s_1^\star \geq \max\{s_2^\star, \ldots, s_n^\star\}$, then $s_2^\star \geq s_1^\star$. It is also easy to see that for the complete network $\boldsymbol{Q}_{ij} = \frac{1}{n-1}$ and all $j \neq i$, if $s_1^\star \geq \max\{s_2^\star, \ldots, s_n^\star\}$, then all other banks will have a higher threshold $s_i^\star \geq s_1^\star$ because of the higher counterparty risk. An inductive argument then shows that $s_1^\star = s_2^\star = \ldots = s_n^\star$. Therefore, the equilibrium is symmetric for a ring or complete network.

Now consider the case of a regular and circulant network. Without loss of generality, it suffices to consider the case where the degree of the regular and circulant network is 2. Suppose that $\boldsymbol{Q}_{i,i+1} = \boldsymbol{Q}_{n,1} = \alpha$ and $\boldsymbol{Q}_{i,i+2} = \boldsymbol{Q}_{n,2} = \boldsymbol{Q}_{n-1,1} = 1-\alpha$. Further, suppose that $s_1^\star \geq \max\{s_2^\star, \ldots, s_n^\star\}$. If $s_n^\star \geq s_{n-1}^\star$, then $s_2^\star \geq s_1^\star \geq \max\{s_2^\star, \ldots, s_n^\star\}$. This is because bank 2's share of the claims on bank 1 is α and its share of the claims on bank n is $1-\alpha$, and, similarly, bank 1's share of the claims in bank n is α and its share of the claims in bank $n-1$ is $1-\alpha$ ($s_1^\star \geq s_n^\star$ and $s_n^\star \geq s_{n-1}^\star$). An inductive argument shows that $s_n^\star \geq s_{n-1}^\star \geq \ldots \geq s_1^\star \geq \max\{s_2^\star, \ldots, s_n^\star\}$.

Note that bank 1's share of the claims on bank n is α and that its share of the claims on bank $n-1$ is $1-\alpha$. Bank n's counterparties are bank $n-1$ and bank $n-2$, with claim shares of α and $1-\alpha$, respectively. In the other case $s_{n-1}^\star \geq s_n^\star$. In order to have $s_1^\star \geq s_n^\star$ we must also have $s_{n-2}^\star \geq s_{n-1}^\star$. An inductive argument then shows that $s_1^\star \geq s_2^\star \geq \ldots \geq s_n^\star$, which contradicts with the fact that the most risky bank, or bank 1, has counterparties which are the least risky banks. Thus, the only possible case is that $s_1^\star = s_2^\star = \ldots = s_n^\star$ and the equilibrium is also symmetric in this case. $\square$

Proof of Proposition 2: In a financial network $(\boldsymbol{Q}, \boldsymbol{y})$, given any residual liquidity $\boldsymbol{e}$, the clearing payment vector $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e})$ is a fixed point of the map Φ, defined by $\Phi(\boldsymbol{x}; \boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) = [\min\{\boldsymbol{y}, \boldsymbol{Q}\boldsymbol{x} + \boldsymbol{e}\}]^+$. Suppose that $\boldsymbol{y'} = \boldsymbol{y} + \Delta\boldsymbol{1}$, where Δ is a positive constant number. The aim is to show that $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) \leq \boldsymbol{x}' = \boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \Delta\boldsymbol{1}$. Now,

$$
\begin{aligned}
\Phi(\boldsymbol{x}'; \boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) &= [\min\{\boldsymbol{y} + \Delta \times \boldsymbol{1}, \boldsymbol{Q}\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \Delta\boldsymbol{1} + \boldsymbol{e}\}]^+ \\
&= \max\{0,\ \min\{\boldsymbol{y}, \boldsymbol{Q}\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \boldsymbol{e}\} + \Delta\boldsymbol{1}\} \\
&\leq \max\{\Delta\boldsymbol{1},\ \min\{\boldsymbol{y}, \boldsymbol{Q}\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \boldsymbol{e}\} + \Delta\boldsymbol{1}\} \\
&= \max\{0,\ \min\{\boldsymbol{y}, \boldsymbol{Q}\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \boldsymbol{e}\}\} + \Delta\boldsymbol{1} \\
&= \boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \Delta\boldsymbol{1} = \boldsymbol{x}'.
\end{aligned}
$$

Since $\Phi(\boldsymbol{x}'; \boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) \leq \boldsymbol{x}'$, it follows that $\boldsymbol{x}(\boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) \leq \boldsymbol{x}'$. Hence, $x_j(\boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) \leq x_j(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \Delta$ for all $j \in \{1, 2, \ldots, n\}$ and $\sum_j x_{ij}(\boldsymbol{Q}, \boldsymbol{y'}, \boldsymbol{e}) \leq \sum_j x_{ij}(\boldsymbol{Q}, \boldsymbol{y}, \boldsymbol{e}) + \Delta$ for all financial networks $(\boldsymbol{Q}, \boldsymbol{y})$

and residual liquidities $\boldsymbol{e}$. Let B be the payoff difference defined by

$$\begin{aligned} B(s^\star,\Delta) \;\; \equiv \;\; & \int_{\boldsymbol{\theta}_{-i}} \frac{r-\sigma}{1+\sigma}\left(s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0) + \Delta\right) - \left(s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0) + 1 + \sigma + \Delta\right) \\ & \times \left[\ln 2 - \ln\left(\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0) + \Delta}{1+\sigma} + 1\right)\right] dJ(\boldsymbol{\theta}_{-i}|s^\star) - \frac{1+r}{1+\sigma}(y+\Delta). \end{aligned}$$

Then, if $s^\star$ is the equilibrium for $(\boldsymbol{Q},\boldsymbol{y})$, equations (B.3) and (B.2) imply that $B(s^\star,0)=0$. Taking derivatives with respect to Δ yields

$$\frac{dB}{d\Delta} = \frac{r-\sigma}{1+\sigma} + 1 - \frac{1+r}{1+\sigma} - \int_{\boldsymbol{\theta}_{-i}} \ln \frac{2}{\frac{s_i^\star + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0) + \Delta}{1+\sigma} + 1} dJ(\boldsymbol{\theta}_{-i}|s^\star) < 0.$$

Consequently, for the new network $(\boldsymbol{Q},\boldsymbol{y}')$,

$$H(s^\star,s^\star) \leq B(s^\star,\Delta) < 0.$$

Hence, at equilibrium, $s^\star(\boldsymbol{Q},\boldsymbol{y}') > s^\star(\boldsymbol{Q},\boldsymbol{y})$. Notice that at the new equilibrium, for any possible $\boldsymbol{e}$, the total repayment of interbank loans for each bank i is given by $\sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y}',\boldsymbol{e}) \leq \sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}) + \Delta$. Now, given the same realization of $\boldsymbol{\theta}$, since $s^\star(\boldsymbol{Q},\boldsymbol{y}') > s^\star(\boldsymbol{Q},\boldsymbol{y})$, it follows that $\boldsymbol{e}(\boldsymbol{\theta}, s^\star(\boldsymbol{Q},\boldsymbol{y}')) < \boldsymbol{e}(\boldsymbol{\theta}, s^\star(\boldsymbol{Q},\boldsymbol{y}))$.[39] Thus,

$$\sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y}',\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}'))) < \sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y}',\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}))) \leq \sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}))) + \Delta.$$

Given any realization of $\boldsymbol{\theta}$, if bank i defaults under the financial network $(\boldsymbol{Q},\boldsymbol{y})$, i.e., $\sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}))) + \theta_i < y + w_i(s^\star(\boldsymbol{Q},\boldsymbol{y}))$, then bank i must default under the new financial network, or

$$\begin{aligned} \sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y}',\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}'))) + \theta_i \;\; &< \;\; \sum_j x_{ij}(\boldsymbol{Q},\boldsymbol{y},\boldsymbol{e}(\boldsymbol{\theta},s^\star(\boldsymbol{Q},\boldsymbol{y}))) + \Delta + \theta_i \\ < y + \Delta + w_i(s^\star(\boldsymbol{Q},\boldsymbol{y})) \;\; &< \;\; y + \Delta + w_i(s^\star(\boldsymbol{Q},\boldsymbol{y}')). \end{aligned}$$

□

[39] For convenience, the network $(\boldsymbol{Q},\boldsymbol{y})$ has been dropped from the notation for $\boldsymbol{e}$.

Proof of Lemma 3: We can see that $(\boldsymbol{Q}^\gamma, \boldsymbol{y}^\gamma)$ is regular according to Definition 1.1 because $\boldsymbol{y}^\gamma = \gamma y_1 \times \mathbf{1} + (1-\gamma) y_2 \times \mathbf{1} = (\gamma y_1 + (1-\gamma) y_2) \times \mathbf{1}$. It is symmetric because for all $i, j \in \mathcal{N}$, and all $k \in \mathcal{N} \setminus \{1\}$,

$$\boldsymbol{Q}^\gamma_{i,i+k} = \gamma \boldsymbol{Q}^1_{i,i+k} + (1-\gamma)\boldsymbol{Q}^2_{i,i+k} = \gamma \boldsymbol{Q}^1_{j,j+k} + (1-\gamma)\boldsymbol{Q}^2_{j,j+k} = \boldsymbol{Q}^\gamma_{j,j+k}.$$

□

Proof of Proposition 3:

(1) First, by Lemma 5,

$$\boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}) \wedge \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}) \le \boldsymbol{x}^\star(\boldsymbol{Q}^\gamma, \boldsymbol{y}, \boldsymbol{e}_\gamma) \le \boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}) \vee \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}).$$

Hence, for each i,

$$\left[\boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}_1) \wedge \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}_2)\right]_i \le \boldsymbol{x}^\star_i(\boldsymbol{Q}^\gamma, \boldsymbol{y}, \boldsymbol{e}_\gamma) \le \left[\boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}_1) \vee \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}_2)\right]_i. \tag{B.8}$$

Suppose that the equilibrium threshold for $(\boldsymbol{Q}^1, \boldsymbol{y})$ is $s_1^\star \equiv s^\star(\boldsymbol{Q}^1, \boldsymbol{y})$ and, for $(\boldsymbol{Q}^2, \boldsymbol{y})$ is $s_2^\star \equiv s^\star(\boldsymbol{Q}^2, \boldsymbol{y})$. Further, suppose that $\boldsymbol{Q}^1$ is equivalent to $\boldsymbol{Q}^2$. Then $s_1^\star = s_2^\star$.

Consider bank i's creditors and suppose that $\{\theta_j\}_{j \neq i}$ is given. Since $s_1^\star = s_2^\star$, the $e_j = \theta_j - w_j(\theta_j, s_j^\star)$ are the same for the two networks. Notice that, for any given $\{e_j\}_{j\neq i}$, there exists $e_i^{1,0}$ and $e_i^{2,0}$ such that the equilibrium clearing payment vector satisfies $x_i(\boldsymbol{e}_1, \boldsymbol{Q}^1) = 0$ and $x_i(\boldsymbol{e}_2, \boldsymbol{Q}^2) = 0$. Here $\boldsymbol{e}_1 = (e_1, e_2, \ldots, e_i^{1,0}, \ldots, e_n)$ and $\boldsymbol{e}_2 = (e_1, e_2, \ldots, e_i^{2,0}, \ldots, e_n)$. Equivalently, $\sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}^1) + e_i^{1,0} = 0$ and $\sum_{j \neq i} x_{ij}(w_0^2, \boldsymbol{Q}^2) + e_i^{2,0} = 0$. Thus,

$$\left[\boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}_1) \wedge \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}_2)\right]_i = \left[\boldsymbol{x}^\star(\boldsymbol{Q}^1, \boldsymbol{y}, \boldsymbol{e}_1) \vee \boldsymbol{x}^\star(\boldsymbol{Q}^2, \boldsymbol{y}, \boldsymbol{e}_2)\right]_i = 0.$$

We can see from equation (B.8) that $\boldsymbol{x}^\star_i(\boldsymbol{Q}^\gamma, \boldsymbol{y}, \boldsymbol{e}_\gamma) = 0$ when $\boldsymbol{e}_\gamma = \gamma \boldsymbol{e}_1 + (1-\gamma)\boldsymbol{e}_2 = (e_1, e_2, \ldots, \gamma e_i^{1,0} + (1-\gamma)e_i^{2,0}, \ldots, e_n)$. Then, it follows that for network $(\boldsymbol{Q}^\gamma, y)$, the sum $\sum_{j\neq i} x_{ij}(w_0^\gamma, \boldsymbol{Q}^\gamma) + \gamma e_i^{1,0} + (1-\gamma)e_i^{2,0} = 0$ and, hence,

$$\sum_{j \neq i} x_{ij}(w_0^\gamma, \boldsymbol{Q}^\gamma) = \gamma \sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}^1) + (1-\gamma) \sum_{j \neq i} x_{ij}(w_0^2, \boldsymbol{Q}^2).$$

Taking convex combinations, if $s^\star_\gamma$ is the equilibrium threshold, then

$$H(s^\star_\gamma, s^\star_\gamma) = \int_{\boldsymbol{\theta}_{-i}} U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^\gamma, \boldsymbol{Q}^\gamma)\right) dJ(\boldsymbol{\theta}_{-i}|s^\star_i) - \frac{1+r}{1+\sigma}y = 0.$$

Since $U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0)\right)$ is a convex function of $\sum_{j\neq i} x_{ij}(w_0)$ and since

$$\frac{d^2U}{d(\sum_{j\neq i} x_{ij}(w_i^0))^2} = \frac{1}{s^\star_i + \frac{1}{2}\sigma + \sum_{j\neq i} x_{ij}(w_i^0)} > 0,$$

it follows that

$$\begin{aligned} U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^\gamma, \boldsymbol{Q}^\gamma)\right) &= U\left(s^\star_\gamma, s^\star_\gamma, \gamma\sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}^1) + (1-\gamma)\sum_{j\neq i} x_{ij}(w_0^2, \boldsymbol{Q}^2)\right) \qquad \text{(B.9)} \\ &< \gamma U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}^1)\right) + (1-\gamma) U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^2, \boldsymbol{Q}^2)\right) \end{aligned}$$

Hence,

$$\begin{aligned} H(s^\star_\gamma, s^\star_\gamma) &= 0 < \gamma \int_{\boldsymbol{\theta}_{-i}} U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}^1)\right) dJ(\boldsymbol{\theta}_{-i}|s^\star_i) \\ &\quad +(1-\gamma)\int_{\boldsymbol{\theta}_{-i}} U\left(s^\star_\gamma, s^\star_\gamma, \sum_{j\neq i} x_{ij}(w_0^2, \boldsymbol{Q}^2)\right) dJ(\boldsymbol{\theta}_{-i}|s^\star_i) - \frac{1+r}{1+\sigma}y. \end{aligned}$$

We have $s^\star_\gamma > s^\star$ since

$$\begin{aligned} &\gamma \int_{\boldsymbol{\theta}_{-i}} U\left(s^\star, s^\star, \sum_{j\neq i} x_{ij}(w_0^1, \boldsymbol{Q}_1)\right) dJ(\boldsymbol{\theta}_{-i}|s^\star_i) \\ &+(1-\gamma)\int_{\boldsymbol{\theta}_{-i}} U\left(s^\star, s^\star, \sum_{j\neq i} x_{ij}(w_0^2, \boldsymbol{Q}_2)\right) dJ(\boldsymbol{\theta}_{-i}|s^\star_i) - \frac{1+r}{1+\sigma}y \quad = \quad 0. \end{aligned}$$

The next step is to show that the less diversified network is less fragile according to Definition 4.1. Without loss of generality, we need only consider the *ex-ante* probability of bankruptcy for bank 1. When two financial networks, $\boldsymbol{Q}^1$, $\boldsymbol{Q}^2$, are equivalent, there exists at least one permutation $\rho : \mathcal{N} \to \mathcal{N}$ that will make them appear identical and keep $\rho(1) = 1$.[40] Let ρ be such a permutation,

[40]Essentially, this is just assigning different numbers to banks in the network. The resulting matrix $\boldsymbol{Q}$ will be

and write $\boldsymbol{Q}_{kl} = \boldsymbol{Q}_{\rho(k),\rho(l)}$ for all $k, l \in N$ and $k \neq l$. For any realization of $\boldsymbol{\theta}(\boldsymbol{Q}^1) = (\theta_1, \theta_2, \ldots, \theta_n)^T$ in network $\boldsymbol{Q}^1$, let the realization of liquidity shocks $\boldsymbol{\theta}(\boldsymbol{Q}^2)$ in $\boldsymbol{Q}^2$ satisfy $\theta_{\rho(i)} = \theta_i$ for all $i \in \mathcal{N}$. Under this transformation, the two networks are identical and bank 1's solvency will be the same in the two different networks.

The inverse function of ρ is denoted by $\nu : \mathcal{N} \to \mathcal{N}$. The function that transforms $\boldsymbol{\theta}(\boldsymbol{Q}^1)$ into $\boldsymbol{\theta}(\boldsymbol{Q}^2)$ is the function $\Psi : [\underline{\theta}, \bar{\theta}]^n \to [\underline{\theta}, \bar{\theta}]^n$ defined by $\Psi(\boldsymbol{\theta}) = (\theta_{\nu(1)}, \theta_{\nu(2)}, \ldots, \theta_{\nu(n)})^T$. Hence, for any $\boldsymbol{\theta}$ in $\boldsymbol{Q}^1$, the problem of having $\Psi(\boldsymbol{\theta})$ in $\boldsymbol{Q}^2$ is exactly the same as the problem of having $\boldsymbol{\theta}$ in $\boldsymbol{Q}^1$. Let the set of all possible liquidity shocks that could make bank 1 default under network $\boldsymbol{Q}^1$ be denoted by $\Theta^1_D \equiv \{\boldsymbol{\theta} | x_1(\boldsymbol{e}(\boldsymbol{\theta}, s^\star), \boldsymbol{Q}^1) < y\}$. Then for any $\boldsymbol{\theta} \in \Theta^1_D$, we can see that $\Psi(\boldsymbol{\theta}) \in \Theta^2_D = \{\boldsymbol{\theta} | x_1(\boldsymbol{e}(\boldsymbol{\theta}, s^\star), \boldsymbol{Q}^2) < y\}$ since the problem for bank 1 is exactly the same in network $\boldsymbol{Q}^2$ with $\Psi(\boldsymbol{\theta})$. The following argument shows that the measure of Θ^γ_D, which is the set of all possible liquidity shocks that would make bank 1 default under network $\boldsymbol{Q}^\gamma$, is larger than the measure of Θ^1_D or Θ^2_D.

For all $\boldsymbol{\theta} \in \Theta^1_D$, we can see that $\boldsymbol{x}_1(\boldsymbol{e}(\boldsymbol{\theta}, s^\star), \boldsymbol{Q}^1) < y$ and, hence, $\boldsymbol{x}_1(\boldsymbol{e}(\Psi(\boldsymbol{\theta}), s^\star), \boldsymbol{Q}^2) < y$. By Lemma 5,

$$\boldsymbol{x}_1(\boldsymbol{e}(\gamma\boldsymbol{\theta} + (1-\gamma)\Psi(\boldsymbol{\theta}), s^\star), \boldsymbol{Q}^\gamma) \leq \boldsymbol{x}_1(\boldsymbol{e}(\boldsymbol{\theta}, s^\star), \boldsymbol{Q}^1) \vee \boldsymbol{x}_1(\boldsymbol{e}(\Psi(\boldsymbol{\theta}), s^\star), \boldsymbol{Q}^2) < y.$$

Since $s^\star_\gamma \geq s^\star$, at equilibrium,

$$\boldsymbol{x}_1(\boldsymbol{e}(\gamma\boldsymbol{\theta} + (1-\gamma)\mathbb{P}(\boldsymbol{\theta}), s^\star_\gamma), \boldsymbol{Q}^\gamma) < \boldsymbol{x}_1(\boldsymbol{e}(\gamma\boldsymbol{\theta} + (1-\gamma)\mathbb{P}(\boldsymbol{\theta}), s^\star), \boldsymbol{Q}^\gamma) < y.$$

Hence, $\Theta^\gamma_D \supseteq \{\gamma\boldsymbol{\theta} + (1-\gamma)\Psi(\boldsymbol{\theta}) | \boldsymbol{\theta} \in \Theta^1_D\}$. Since Ψ is a one-to-one mapping, the measure of Θ^γ_D is greater than the measure of Θ^1_D, i.e., $m(\Theta^\gamma_D) \geq m(\Theta^1_D)$. By Definition 4.1, $\boldsymbol{Q}^\gamma$ is more fragile than $\boldsymbol{Q}^1$ (or, equivalently, $\boldsymbol{Q}^2$).

(2) It is easy to see that the ring network, i.e., $\boldsymbol{Q}_{i,i+1} = \boldsymbol{Q}_{n,1} = 1$, cannot be a convex combination of symmetric and regular networks. The next step is to show that the complete network, i.e., $\boldsymbol{Q}_{ij} = \frac{1}{n-1}$ for $j \neq i$ can be written as a convex combination of a symmetric network $\boldsymbol{Q}$ and its equivalent networks.

Without loss of generality, we need only consider the first row of the relative liability matrix, given that the first row of $\boldsymbol{Q}$ is $I_1 \times \boldsymbol{Q} = [\boldsymbol{Q}_{11} = 0, \boldsymbol{Q}_{12}, \ldots, \boldsymbol{Q}_{1n}]$, where $I_1 = [1, 0, 0, \ldots 0]$. An

different, but there is no real effect.

equivalent network can be constructed by taking $I_1\boldsymbol{Q}^1 = [\boldsymbol{Q}_{11} = 0, \boldsymbol{Q}_{13}, \boldsymbol{Q}_{14} \ldots, \boldsymbol{Q}_{1n}, \boldsymbol{Q}_{12}]$, $I_1\boldsymbol{Q}^2 = [\boldsymbol{Q}_{11} = 0, \boldsymbol{Q}_{14}, \boldsymbol{Q}_{15} \ldots, \boldsymbol{Q}_{12}, \boldsymbol{Q}_{13}], \ldots,$ $I_1\boldsymbol{Q}^{n-2} = [\boldsymbol{Q}_{11} = 0, \boldsymbol{Q}_{1n}, \boldsymbol{Q}_{12} \ldots, \boldsymbol{Q}_{1,n-1}]$. The remaining parts of the relative liability matrix will change accordingly.[41]. It is then easy to see that the complete network can be written as the convex combination $\boldsymbol{Q}^C = \frac{1}{n-1}\boldsymbol{Q} + \frac{1}{n-1}\boldsymbol{Q}^1 + \ldots + \frac{1}{n-1}\boldsymbol{Q}^{n-2}$

(3) The following argument shows that for any financial network in $\mathbb{Q}_m$, the financial network $\boldsymbol{Q}_0^m = \{\boldsymbol{Q} \in \mathbb{Q}_m | D(\boldsymbol{Q}) = \{\frac{1}{m}\}\}$ of degree m is more diversified than any $\boldsymbol{Q} \in \mathbb{Q}_m$ and any $\boldsymbol{Q} \notin \boldsymbol{Q}_0^m$. For any $\boldsymbol{Q} \in \mathbb{Q}_m$, each bank has m connections. Adopting a similar method to the one used to prove part (2) for the complete network, we can find $m-1$ symmetric networks equivalent to $\boldsymbol{Q}$ in which each bank is connected to the same set of banks, but different weights are assigned to each bank. Thus, $\boldsymbol{Q}_0^m$ is a convex combination of these m equivalent networks. $\square$

Proof of Proposition 4: Given the information $s_i = \theta_i + \sigma\epsilon_i$, bank j's liquidity, $\theta_j = s_i - \sigma\epsilon_i + (1-\mu)(\eta_j - \eta_i)$ will be distributed as

$$\theta_j | s_i \sim N(s_i, \sigma^2 + 2(1-\mu)^2\sigma_\eta^2).$$

Note that $U(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0)$ is a convex function of $\sum_{j\neq i} x_{ij}^0$ and $\sum_{j\neq i} x_{ij}^0$ is weakly increasing in each θ_j for $j \neq i$. It is easy to show that a mean-preserving spread of the distribution of θ_j (given all other θ_k, $k \neq j$) would give rise to a mean-preserving spread of $\sum_{j\neq i} x_{ij}^0$ and lower the value of the indifference function $H = \mathbb{E}_{F(\theta_j|s_i^\star)}\left[U(s^\star, s^\star, \sum_{j\neq i} x_{ij}^0)\right]$.

The normal distribution $N(s_i, \sigma^2 + 2(1-\mu_2)^2\sigma_\eta^2)$ is a mean-preserving spread of $N(s_i, \sigma^2 + 2(1-\mu_1)^2\sigma_\eta^2)$ if $\mu_1 > \mu_2$. Thus, when $\mu_1 > \mu_2$, we see that $s^\star(\mu_1) > s^\star(\mu_2)$. For a fixed financial network $(\boldsymbol{Q}, \boldsymbol{y})$, it is evident that more panics from creditors with higher μ-values would make the financial network more fragile. $\square$

C Proofs for Core Periphery Networks

Proof of Proposition 5: I show the existence and uniqueness of the symmetric equilibrium for $(s_c^\star, s_p^\star)$. Suppose that all other core bank creditors' strategies $s_c^\star$ and all periphery bank creditors'

[41] Note that the constant defined in Equation (B.4) is positive when the network is complete. However, it is easy to check that the maximum value of the constant is dominated by the difference in Equation (B.9) when $y < y_0$ and $n \geq 2$.

strategies $s_p^\star$ are known. Let $H_c(s_c^\star, s_p^\star)$ be the expected payoff difference for the core bank creditor c with information $s_c^\star$, and let $H_p(s_p^\star, s_c^\star)$ be the expected payoff difference for the peripheral bank creditor p with information $s_p^\star$. The following argument shows that there is a unique pair $(s_c^\star, s_p^\star)$ which satisfies the equilibrium condition: $H(s_c^\star, s_p^\star) = H(s_p^\star, s_c^\star) = 0$.

First, the monotonicity of $H_c(s_c, s_p)$ in s_c and s_p is demonstrated. Suppose that s_p is the equilibrium threshold for periphery banks' creditors, and consider any bank $i \in \mathcal{C}$. Suppose that $s_c^0 = s_c + \eta$, where $\eta > 0$ is a small constant close to zero. Note that for all other core banks $j \in \mathcal{C}$, given any realization of $(\theta_j)_{j \neq i}$, since $e_j = \theta_j - F(\frac{s_j - \theta_j}{\sigma}) = \frac{1+\sigma}{\sigma}\theta_j - \frac{s_j + \frac{\sigma}{2}}{\sigma}$, we see that $e_j(s, \theta_j) = e_j(s_0, \theta_j) + \frac{\eta}{\sigma}$.[42]

Consider the case where $\theta_j^{'} = \theta_j + \frac{1}{1+\sigma}\eta$ for all $j \in \mathcal{C}$ satisfying $j \neq i$, and $e_j(s^\star, \theta_j) = e_j(s_0, \theta_j^{'})$. Define $\theta_i^0(s_i, \{\theta_j\}_{j \neq i})$ to be the threshold liquidity such that $x_i = 0$ and $e_i^0 = \frac{1+\sigma}{\sigma}\theta_i^0 - \frac{s_i + \frac{\sigma}{2}}{\sigma}$. Under this transformation, all interbank payments $\{x_j\}_{j \neq i}$(including the payments from periphery banks) are identical. By a similar argument to the one given in the proof of Proposition 1, $H_c(s_c^0, s_p) > H_c(s_c, s_p)$. It is easy to see that $H_c(s_c, s_p)$ is decreasing in s_p since an increase in s_p will weakly decrease the interbank payment $\sum_{j \neq i} x_{ij}^0$. Since $H_c(s_c, s_p)$ is monotone in s_c and s_p , there exists a one-to-one mapping $\Lambda_1 : [\underline{s}, \bar{s}] \to [\underline{s}, \bar{s}]$ such that $\Lambda_1(s_p) = s_c$. All possible equilibriums for $H(s_c, s_p) = 0$ can be written in the form $(\Lambda_1(s_p),\ s_p)$ for $s_p^\star \in [\underline{s}, \bar{s}]$. By applying the same argument to the problem for creditors of periphery banks, we can see that there exists a one-to-one mapping $\Lambda_2 : [\underline{s}, \bar{s}] \to [\underline{s}, \bar{s}]$ such that $\Lambda_2(s_c) = s_p$ for all possible solutions of the equation $H_p(s_p, s_c) = 0$. The symmetric equilibrium is then a pair $(s_c^\star, s_p^\star)$ such that $\Lambda_1(s_p^\star) = s_c^\star$ and $\Lambda_2(s_c^\star) = s_p^\star$.

For any positive number η, the argument used to derive Equation (B.7) shows that $H_c(s_c + \eta, s_p + \eta) > H_c(s_c, s_p)$ and $H_p(s_p + \eta, s_c + \eta) > H_p(s_p, s_c)$. Hence, there exist constants $0 < \beta_1 < 1$ and $0 < \beta_2 < 1$ and a strategy s_p such that $\Lambda_1(s_p + \eta) < \Lambda_1(s_p) + \beta_1\eta$ and $\Lambda_2(s_c + \eta) < \Lambda_2(s_c) + \beta_2\eta$ for all $\eta > 0$. Let $\Lambda : [\underline{s}, \bar{s}] \to [\underline{s}, \bar{s}]$ be the one-to-one mapping such that $\Lambda \equiv \Lambda_1 \circ \Lambda_2$. Then

$$\Lambda(s_c + \eta) = \Lambda_1(\Lambda_2(s_c + \eta)) < \Lambda_1(\Lambda_2(s_c) + \beta_1\eta) < \Lambda(s_c) + \beta_1\beta_2\eta.$$

By Banach's fixed-point theorem, there exists a unique $s_c^\star \in \bar{\mathbb{S}}$ such that $\Lambda(s_c^\star) = s_c^\star$. The unique equilibrium is then $(s_c^\star, s_p^\star = \Lambda_2(s_c^\star))$. □

Proof of Proposition 6: Suppose that each core bank $i \in \mathcal{C}$ has liquid assets of $1 + \theta_i + \sum_{j \in \mathcal{P}} x_{ij} +$

[42]The realization of $\{\theta_k\}_{k \in \mathcal{P}}$ stays the same and, for simplicity, will be omitted from this part of the proof.

$\sum_{k\in\mathcal{C}} x_{ik}$, and a liability of $1 + w_i + qy_p + y_c$. Moreover, suppose that each periphery bank $u \in \mathcal{P}$ has liquid asset holdings of $1 + \theta_u + x_{i,o(u)}$ and a total liability of $1 + w_u + y_p$.[43] Note that, in the symmetric equilibrium, all creditors of core banks have the same strategy by Proposition 5. Hence, if $y_c = y_p$ in expectation, then $\sum_{k\in\mathcal{C}} x_{ik}$ is the same as $x_{u,o(u)}$, since $\sum_{k\in\mathcal{C}} x_{ik}$ is just a weighted average of all payments (with face value y_c) from connected core banks, while $x_{i,o(u)}$ is the payment from one of these banks, p, with face value y_p.

By Proposition 3, the mean spread of the distribution of payments ($\sum_{k\in\mathcal{C}} x_{ik}$ *v.s.* $x_{i,o(u)}$) will make bank $i \in \mathcal{C}$ face more risk from their creditors than bank $u \in \mathcal{P}$. It is evident that bank i faces more counterparty risk since $\sum_{j\in\mathcal{P}} x_{ij} < qy_p$. Thus, when $y_c = y_p$, we see that $s_c^\star > s_p^\star$. If $y_c > y_p$, then the counterparty risk from core banks for bank i, i.e., $y_c - \sum_{k\in\mathcal{C}} x_{ik}$, is higher than that for bank u, i.e., $y_p - x_{u,o(u)}$, which only strengthens the argument. □

Proof of Proposition 7: After normalization, each core bank $i \in \mathcal{C}$ has liquid asset holdings of $1 + \theta_i + \frac{1}{k_c}\sum_{j\in\mathcal{P}} x_{ij} + \frac{1}{k_c}\sum_{k\in\mathcal{C}} x_{ik}$, and a total liability of $1 + w_i + \frac{\lambda_p}{k_c} p y_p + \frac{\lambda_c}{k_c} y_c$. Each periphery bank $u \in \mathcal{P}$ has liquid asset holdings of $1 + \theta_u + \frac{1}{k_p} x_{i,o(u)}$ and a total liability of $1 + w_u + \frac{\lambda_p}{k_p} y_p$. By Proposition 5, when $\frac{\lambda_p}{k_p} y_p \leq \frac{\lambda_c}{k_c} y_c$ the core banks will face higher risk than the periphery banks.

By Proposition 2, it is easy to see that, *ceteris paribus*, an increase in $\lambda_p(\lambda_c)$ or a decrease in $k_p(k_c)$ will increase the total interbank lending $\frac{\lambda_p}{k_p} y_p$ (that is, $\frac{\lambda_p}{k_c} p y_p + \frac{\lambda_c}{k_c} y_c$), which will make the system more fragile.

Increasing k_c, λ_c and λ_p while keeping $\frac{\lambda_p}{k_c}$ and $\frac{\lambda_c}{k_c}$ constant will not change the risk faced by creditors of core banks, i.e., $s_c^\star$ will stay constant if $s_p^\star$ stays constant. However, the resulting increase in $\frac{\lambda_p}{k_p}$ will raise $s_p^\star$ and $s_c^\star$ will also increase. This means that the system is more fragile as both core banks and periphery banks will have higher risks of defaulting. □

D Proofs for the Distressed Case

Proof of Proposition 8: Note that each creditor still faces exactly the same problem *ex-ante*. For bank i's creditors, $\theta_i \sim U[\underline{\theta}, \bar{\theta}]$ and their belief about the other banks' liquidity is given by

$$\theta_j = \begin{cases} \theta_C & \text{w.p} \frac{1}{n-1} \\ \sim U[\underline{\theta}, \bar{\theta}] & \text{w.p.} \frac{n-2}{n-1}. \end{cases}$$

[43] For a periphery bank u, let $o(u) \in \mathcal{C}$ denote its sole debtor bank.

Parts 1., 2. and 3. of this result follow immediately from Propositions 1, 2, and 3, respectively. $\square$

Proof of Proposition 9: In the ring network, for all $i \in \mathcal{N}$, if all creditors of bank i believe that the distressed bank is bank $i-1 \pmod n$, then $x_{i-1} = 0$. Let $(\boldsymbol{Q}, \boldsymbol{y})$ be a financial network in any possible equilibrium, under some realization of $\boldsymbol{\theta}$. Then $\sum_{j \neq i} x_{ij} > 0$. Hence, setting $x_{i-1} = 0$ will give the highest $s^\star$ among all possible symmetric and regular networks. An argument similar to the one used to prove financial fragility in Proposition 3 shows that the ring network is the most fragile one. $\square$

Proof of Proposition 10: For banks in ring networks, the liquid asset holdings are $1 + \theta_i + x_{i-1}$ and the total liability is $1 + y + w_i$.[44] Thus, the equilibrium $s^\star$ is determined by the following equation

$$H(s_i^\star, s_i^\star) = \int_{\boldsymbol{\theta}_{-i}} U(s_i^\star) dJ(\boldsymbol{\theta}_{-i}|s_i^\star) - \frac{1+r}{1+\sigma} y = 0,$$

where

$$U(s_i^\star) = \frac{r-\sigma}{1+\sigma}(s_i^\star + \frac{1}{2}\sigma + x_{i-1}(w_i^0)) - (s_i^\star + \frac{1}{2}\sigma + x_{i-1}(w_i^0) + 1 + \sigma)\left[\ln 2 - \ln(\frac{s_i^\star + \frac{1}{2}\sigma + x_{i-1}(w_i^0)}{1+\sigma} + 1)\right].$$

Note that

$$x_{i-1}(w^0) = \min\{\theta_{i-1} - w(s_{i-1}^\star, \theta_{i-1}), y\}^+ = \min\left\{\left(\frac{1+\sigma}{\sigma}\right)\theta_{i-1} - \frac{s_{i-1}^\star + \frac{1}{2}\sigma}{\sigma},\ y\right\}^+.$$

Abusing the notation, let $H(s_i^\star, s_i^\star, s_{i-1}^\star)$ denote the expected payoff difference for the marginal creditor with information $s_i^\star$, taking bank i's other creditors' strategy $s_i^\star$ and bank $i-1$'s creditors' strategy $s_{i-1}^\star$ into account. Then $H(s_i^\star, s_i^\star, s_{i-1}^\star)$ is a strictly decreasing function of $s_{i-1}^\star$ since, with a certain realization of θ_i, we may write $x_{i-1} = (\frac{1+\sigma}{\sigma})\theta_{i-1} - \frac{s_{i-1}^\star + \frac{1}{2}\sigma}{\sigma} \in (0, y)$. By the monotonicity of $H(s_i^\star, s_i^\star, s_{i-1}^\star)$ in $s_i^\star$, there is a one-to-one mapping $I : [\underline{s}, \bar{s}] \to [\underline{s}, \bar{s}]$ such that $s_i^\star = I(s_{i-1}^\star)$ and $I(.)$ is a strictly increasing function. The condition, $x_1 = 0$ is independent of the realization of θ_1. Equivalently, $s_1^\star = \bar{s} = \bar{\theta} + \frac{\sigma}{2}$, which is the highest possible private information value and this will make $x_1 = 0$ for any possible θ_1. Since $y + 1 < \bar{\theta}$, bank 2's creditors will not withdraw early even if all other creditors withdraw when $s_2 = \bar{s}$. Thus, $s_2^\star = I(s_1^\star) < s_1^\star = \bar{s}$. An inductive argument then

[44] Note that $x_{i-1} = x_{i,i-1}$ since the bank i is the sole creditor bank of bank $i-1$.

shows that $s_i^\star < s_{i-1}^\star$.

Note that $I(.)$ is a mapping from the closed, bounded set $[\underline{s}, \bar{s}]$ to $[\underline{s}, \bar{s}]$ which satisfies $I(\bar{s}) < \bar{s}$ and $I(\underline{s}) > \underline{s}$. By Brouwer's fixed point theorem, there exists at least one fixed point $s^\star \in [\underline{s}, \bar{s}]$ such that $I(s^\star) = s^\star$. A similar argument to the one given in proof of Proposition 1 shows that there exists a unique symmetric equilibrium for the ring network. Thus, the fixed point $s_R^\star$ must be unique. This $s_R^\star$ is the threshold for the ring network when there is no additional shock and $\theta \sim U[\underline{\theta}, \bar{\theta}]$. The decreasing sequence $\{s_i^\star\}_{i=1}^n$ will converge to the fixed point $s_R^\star$ when $n \to \infty$.

By Proposition 3, the equilibrium threshold of the complete network $s_C^\star$ is greater than that of the ring network $s_R^\star$. When there is a defaulting bank in the network *ex-ante*, it is evident that the equilibrium threshold of the complete network will satisfy $s_{CD}^\star > s_C^\star > s_R^\star$. Now, $\{s_i^\star\}_{i=1}^n$ is a decreasing sequence with $s_1 = \bar{s} > s_{CD}^\star$ that converges to s_0. Thus, there exists n_0 such that $n < n_0$ implies that $s_i^\star > s_{CD}^\star$ for all $i \leq n_0$, and, hence, the ring network is more fragile. □

Proof of Proposition 11:

(1) Let $s_{RN}^\star$ be the equilibrium threshold in the ring network when creditors hold natural beliefs and the information about the distressed bank is not available. Let $s_{RC}^\star$ be the equilibrium threshold when creditors hold cautious beliefs about the location of the distressed bank. Let $s_i^\star$ be the equilibrium threshold for bank i when bank 1 is the distressed bank. Given that $\{s_i^\star\}_{i=1}^n$ is a decreasing sequence from $s_1 = \bar{s}$, converging to $s_R^\star$, there exists n_1 for which $n \leq n_1$ implies that $s_i^\star \geq s_{RN}^\star$. Bank i will go bankrupt if $\theta_i + x_{i-1} - w_i < y$. The *ex-ante* probability of a default for bank i is the measure of the set

$$\mathbb{D}(i) \equiv \left(\theta_i \in [\underline{\theta}, \bar{\theta}], \theta_{i-1} \in [\underline{\theta}, \bar{\theta}] \mid \left(\frac{1+\sigma}{\sigma}\right)\theta_i + \min\left\{ \left(\frac{1+\sigma}{\sigma}\right)\theta_{i-1} - \frac{s_{i-1}^\star + \frac{\sigma}{2}}{\sigma},\ y \right\}^+ - \frac{s_i^\star + \frac{\sigma}{2}}{\sigma} < y \right).$$

Because $s_{i-1}^\star > s_i^\star > s_{RN}^\star$, the probability of a default is higher for each bank $i \leq n$ in the network when compared to the case in which this information is not available. Now, $n_1 \geq n_0$ because $s_{RN}^\star < s_{CD}^\star$ (by Proposition 8).

(2) This is evident because when the information about a distressed bank is not available, bank i's creditors will believe that $x_{i-1} = 0$ for all $i \in \mathcal{N}$. When such information is available, all creditors will have rational expectations and only bank 2's creditors will believe that $x_1 = 0$. □

Proof of Proposition 12: In this proof, whenever there are multiple equilibria, the worst possible equilibrium will be considered to be the equilibrium in which the creditors will withdraw early when they expect that all others will do so, even if $\theta_i = \bar{\theta}$ for all $i \in \{2, 3, \dots, n\}$.

(1) In a ring network, for all $i \in \mathcal{N}/\{n\}$, the conditions $\boldsymbol{Q}_{i,i-1} = \boldsymbol{Q}_{n,1} = 1$ and $\boldsymbol{Q}_{ij} = 0$ when $j \neq i$ and $j \neq i-1$ are satisfied. For bank 2, the total asset holding is $1 + \bar{\theta}$ and the total liability is $1 + w + y$. Suppose that all other creditors will withdraw early. Then $w = 1$. Withdrawing is the dominant strategy under this belief since bank 2 must default when $y > \bar{\theta} - 1$, or $\bar{\theta} + 1 < 2 + y$. This is a self-fulfilling equilibrium if each creditor believes that all others will withdraw early. For bank 3, the total asset holding is $1 + \bar{\theta} + (\bar{\theta} - 1)$ and the total liability is $2 + y$. Hence, when $y > 2(\bar{\theta} - 1)$, all creditors withdrawing early, independent of their private information, is an equilibrium. Note that for each bank $i \geq 2$, if the liquidity is $1 + \bar{\theta}$ and all creditors withdraw, then the interbank payment is $x_i = \bar{\theta} - 1$. A straight forward inductive argument shows that $m = [\frac{y}{\bar{\theta}-1}]$.

(2) In a complete network, $\boldsymbol{Q}_{ij} = \frac{1}{n-1}$ for all $i \neq j$. The symmetry of such a network ensures that if the situation of all banks running with complete panic forms an equilibrium, then for each bank i, the equality $1 + \bar{\theta} + \frac{n-2}{n-1}x = 2 + x$ is satisfied. In this equilibrium, all creditors will withdraw early, or $w_i = 1$, and each bank will be able to pay out x to its creditor banks. The creditors' decisions are rational only if $x < y$, in which case the bank cannot completely fulfill its obligations. Hence, this is an equilibrium only if $y > (n-1)(\bar{\theta} - 1)$. If $y \leq (n-1)(\bar{\theta} - 1)$, then no bank will default, independent of the liquidity shocks.

(3) This follows immediately from (1) and (2). □

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